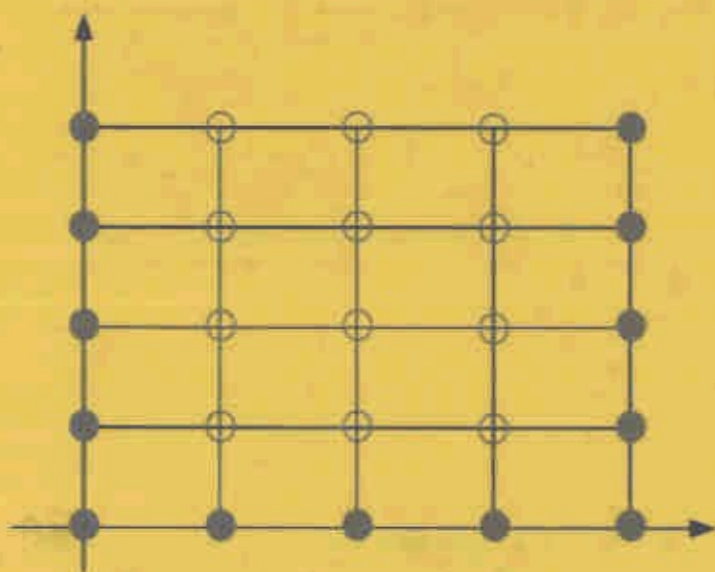


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TEXTS IN APPLIED MATHEMATICS

Introduction to Numerical Methods in Differential Equations



Springer

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