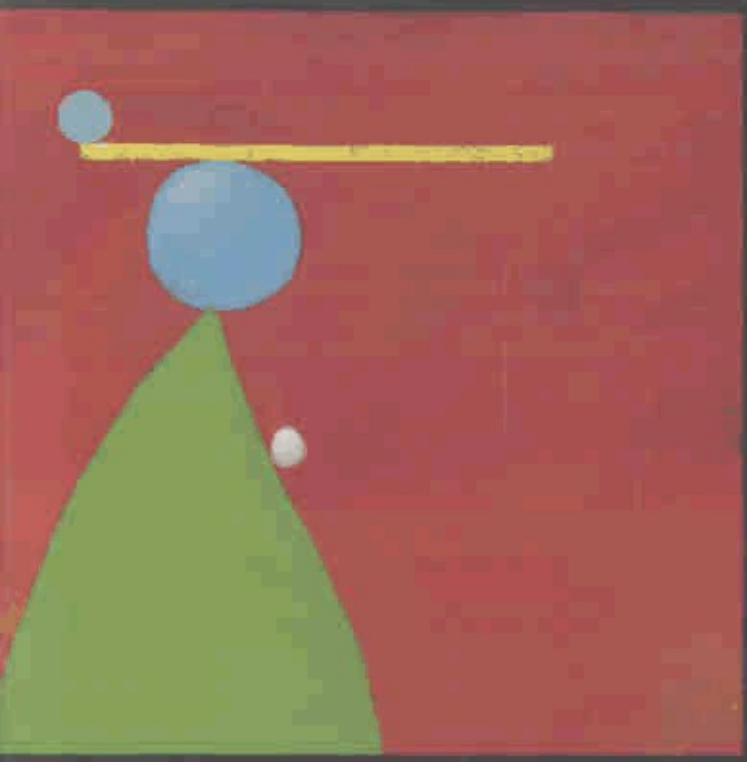
QUANTITATIVE RISK MANAGEMENT



Concepts

Techniques

Tools

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PRINCETON SERIES IN FINANCE

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