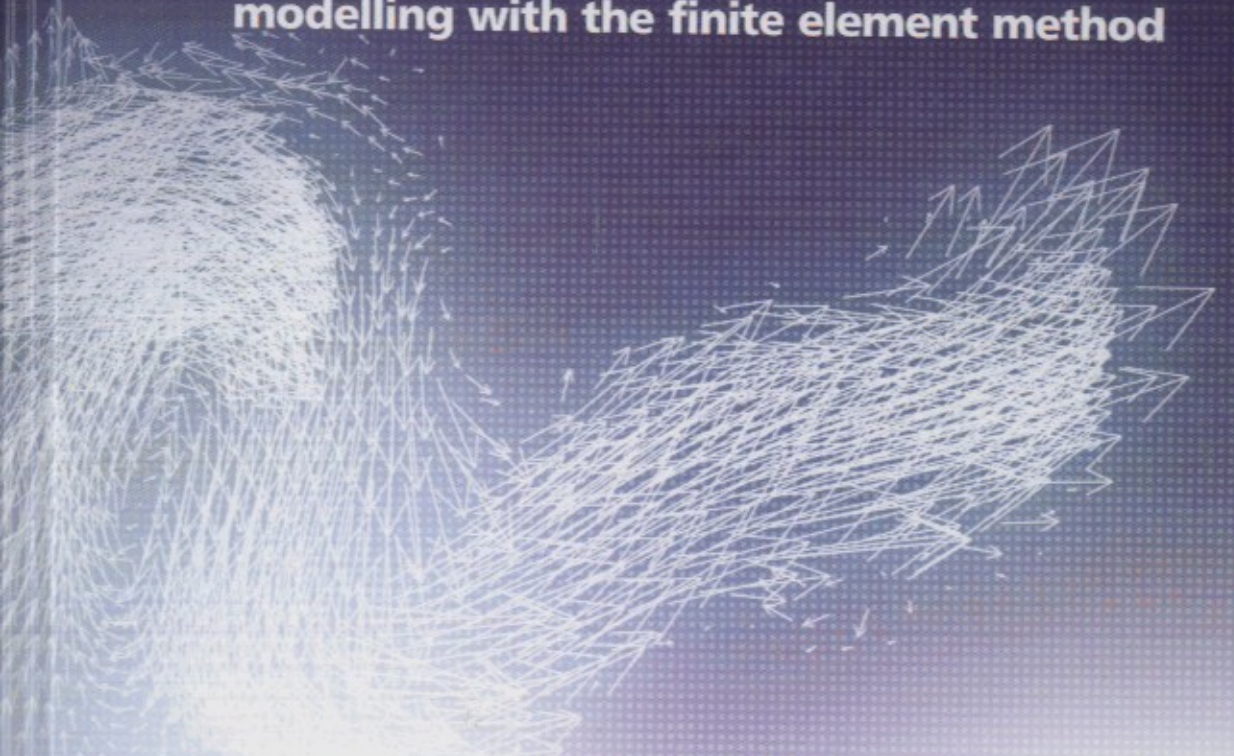


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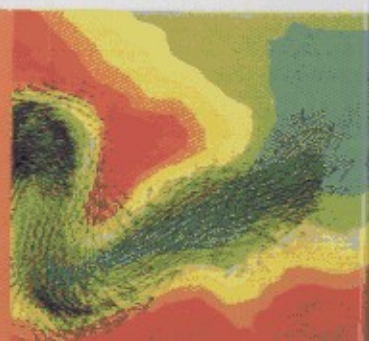
Free Surface Flows

modelling with the finite element method



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