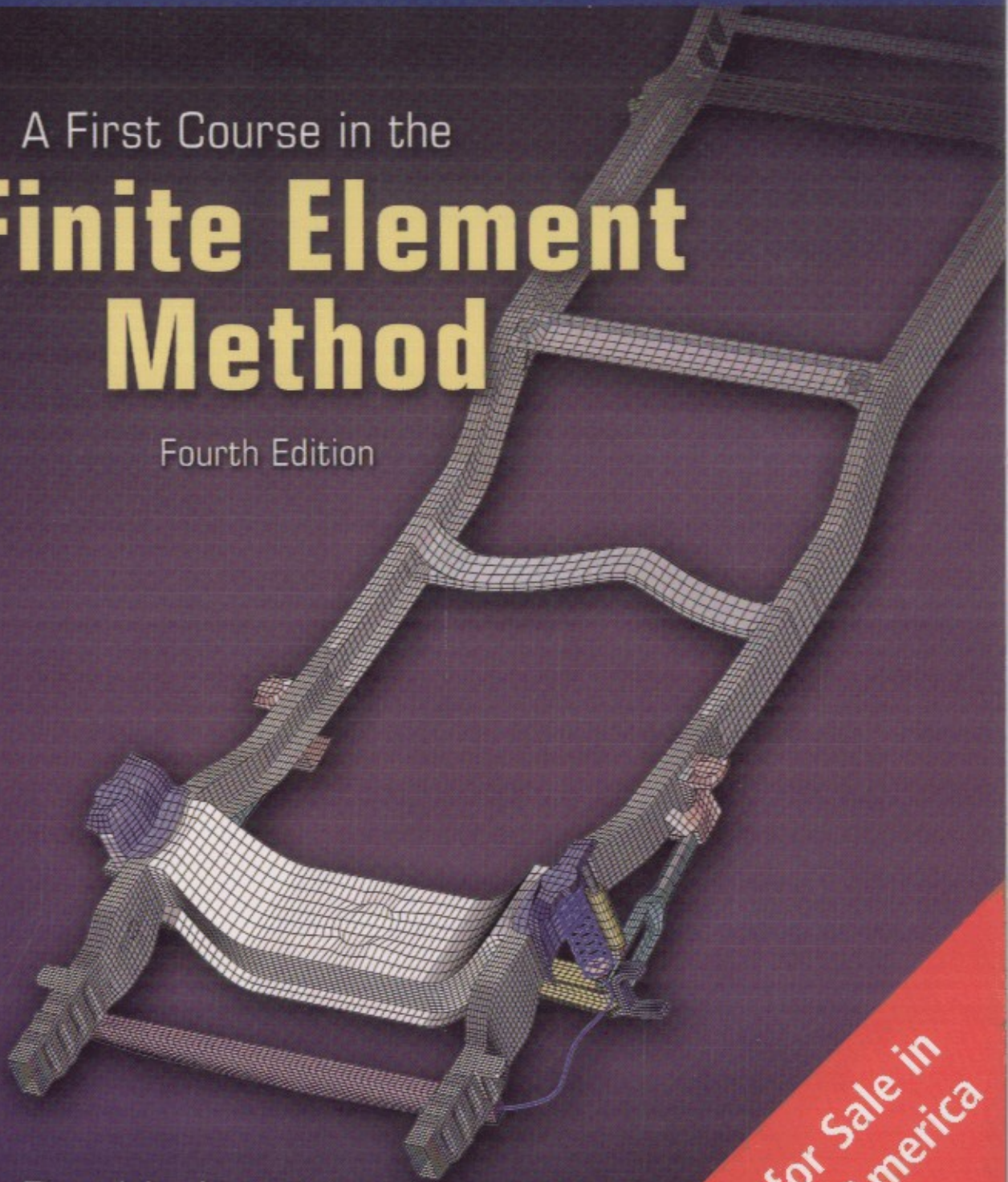


INTERNATIONAL STUDENT EDITION

A First Course in the

# Finite Element Method

Fourth Edition



Daryl L. Logan

Not for Sale in  
North America

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