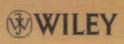
Jacob Fish • Ted Belytschko

A First Course in

## Finite Elements





## **Contents**

Preface			xi
1	Introduction		1
	1.1	Background	1
	1.2	Applications of Finite elements	7
		References	9
2	Direct Approach for Discrete Systems		11
	2.1	Describing the Behavior of a Single Bar Element	11
	2.2	Equations for a System	15
		2.2.1 Equations for Assembly	18
		2.2.2 Boundary Conditions and System Solution	20
	2.3	Applications to Other Linear Systems	24
	2.4	Two-Dimensional Truss Systems	27
	2.5	Transformation Law	30
	2.6	Three-Dimensional Truss Systems	35
		References	36
		Problems	37
3	Stro	ong and Weak Forms for One-Dimensional Problems	41
	3.1	The Strong Form in One-Dimensional Problems	42
		3.1.1 The Strong Form for an Axially Loaded Elastic Bar	42
		3.1.2 The Strong Form for Heat Conduction in One Dimension	44
		3.1.3 Diffusion in One Dimension	46
	3.2	The Weak Form in One Dimension	47
	3.3	Continuity	
	3.4	4 The Equivalence Between the Weak and Strong Forms	
	3.5	One-Dimensional Stress Analysis with Arbitrary Boundary Conditions	58
		3.5.1 Strong Form for One-Dimensional Stress Analysis	58
		3.5.2 Weak Form for One-Dimensional Stress Analysis	59

## vi CONTENTS

	3.6	One-Dimensional Heat Conduction with Arbitrary	
		Boundary Conditions	60
		3.6.1 Strong Form for Heat Conduction in One Dimension	
		with Arbitrary Boundary Conditions	60
		3.6.2 Weak Form for Heat Conduction in One Dimension	
		with Arbitrary Boundary Conditions	61
	3.7	Two-Point Boundary Value Problem with	
		Generalized Boundary Conditions	62
		3.7.1 Strong Form for Two-Point Boundary Value Problems with Generalized Boundary Conditions	62
		3.7.2 Weak Form for Two-Point Boundary Value Problems	02
		with Generalized Boundary Conditions	63
	3.8	Advection-Diffusion	64
		3.8.1 Strong Form of Advection-Diffusion Equation	65
		3.8.2 Weak Form of Advection-Diffusion Equation	66
	3.9	Minimum Potential Energy	67
		Integrability	71
		References	72
		Problems	72
4		proximation of Trial Solutions, Weight Functions	77.
	and	Gauss Quadrature for One-Dimensional Problems	77
	4.1	Two-Node Linear Element	79
	4.2	Quadratic One-Dimensional Element	81
	4.3	Direct Construction of Shape Functions in One Dimension	82
	4.4	Approximation of the Weight Functions	84
	4.5	Global Approximation and Continuity	84
	4.6	Gauss Quadrature	85
		Reference	90
		Problems	90
5	Finite Element Formulation for One-Dimensional Problems		
	5.1	Development of Discrete Equation: Simple Case	93
	5.2	Element Matrices for Two-Node Element	97
	5.3	Application to Heat Conduction and Diffusion Problems	99
	5.4	Development of Discrete Equations for Arbitrary Boundary	
		Conditions	105
	5.5	Two-Point Boundary Value Problem with	
		Generalized Boundary Conditions	111
	5.6	Convergence of the FEM	113
		5.6.1 Convergence by Numerical Experiments	115
		5.6.2 Convergence by Analysis	118
	5.7	FEM for Advection-Diffusion Equation	120
		References	122
		Problems	123

			CONTENTS	vii
6	Stro	ong and Weak Forms for Multidimensional		
		lar Field Problems		131
	6.1	Divergence Theorem and Green's Formula		133
	6.2	Strong Form		139
	6.3	Weak Form		142
	6.4	The Equivalence Between Weak and Strong Forms		144
	6.5	Generalization to Three-Dimensional Problems		145
	6.6	Strong and Weak Forms of Scalar Steady-State		
		Advection-Diffusion in Two Dimensions		146
		References		148
		Problems		148
7		proximations of Trial Solutions, Weight Functions and		
	Gau	ss Quadrature for Multidimensional Problems		151
	7.1	Completeness and Continuity		152
	7.2	Three-Node Triangular Element		154
		7.2.1 Global Approximation and Continuity		157
		7.2.2 Higher Order Triangular Elements		159
		7.2.3 Derivatives of Shape Functions for the Three-Node Triangular Element		160
	7.3	_		161
	7.4			164
		7.4.1 Continuity of Isoparametric Elements		166
		7.4.2 Derivatives of Isoparametric Shape Functions		166
	7.5	Higher Order Quadrilateral Elements		168
	7.6	Triangular Coordinates		172
		7.6.1 Linear Triangular Element		172
		7.6.2 Isoparametric Triangular Elements		174
		7.6.3 Cubic Element		175
		7.6.4 Triangular Elements by Collapsing Quadrilateral Eleme	ents	176
	7.7	Completeness of Isoparametric Elements		177
	7.8	Gauss Quadrature in Two Dimensions		178
		7.8.1 Integration Over Quadrilateral Elements		179
		7.8.2 Integration Over Triangular Elements		180
	7.9	Three-Dimensional Elements		181
		7.9.1 Hexahedral Elements		181
		7.9.2 Tetrahedral Elements		183
		References		185
		Problems		186
8	Finite Element Formulation for Multidimensional			
	Scal	lar Field Problems		189
	8.1	Finite Element Formulation for Two-Dimensional		
	0.2	Heat Conduction Problems		189
	8.2	Verification and Validation		201

viii	CON.	<b>TENTS</b>

	8.3	Advection-Diffusion Equation	207
		References	209
		Problems	209
9	Finite	e Element Formulation for Vector Field Problems – Linear Elasticity	215
	9.1	Linear Elasticity	215
		9.1.1 Kinematics	217
		9.1.2 Stress and Traction	219
		9.1.3 Equilibrium	220
		9.1.4 Constitutive Equation	222
	9.2	Strong and Weak Forms	223
	9.3	Finite Element Discretization	225
	9.4	Three-Node Triangular Element	228
		9.4.1 Element Body Force Matrix	229
		9.4.2 Boundary Force Matrix	230
	9.5	Generalization of Boundary Conditions	231
	9.6	Discussion	239
	9.7	Linear Elasticity Equations in Three Dimensions	240
		Problems	241
10	Finite Element Formulation for Beams		
	10.1	Governing Equations of the Beam	249
		10.1.1 Kinematics of Beam	249
		10.1.2 Stress–Strain Law	252
		10.1.3 Equilibrium	253
		10.1.4 Boundary Conditions	254
	10.2	Strong Form to Weak Form	255
		10.2.1 Weak Form to Strong Form	257
	10.3	Finite Element Discretization	258
		10.3.1 Trial Solution and Weight Function Approximations	258
		10.3.2 Discrete Equations	260
	10.4	Theorem of Minimum Potential Energy	261
	10.5	Remarks on Shell Elements	265
		Reference	269
		Problems	269
11	Com	mercial Finite Element Program ABAQUS Tutorials	275
	11.1	Introduction	275
		11.1.1 Steady-State Heat Flow Example	275
	11.2	Preliminaries	275
	11.3	Creating a Part	276
	11.4	Creating a Material Definition	278
	11.5	Defining and Assigning Section Properties	279
	11.6	Assembling the Model	280
	11.7	Configuring the Analysis	280
	11.8	Applying a Boundary Condition and a Load to the Model	280
	11.9	Meshing the Model	282

		CONTENTS	ix
11.5	0 Creating and Submitting an Analysis Job		284
11.			284
11.1			284
11.1			285
	11.13.1 Bending of a Short Cantilever Beam		287
11.1	4 Copying the Model		287
11.1			287
11.	6 Configuring the Analysis		287
11.1	** * *		
	the Model		288
11.1			289
11.1	9 Creating and Submitting an Analysis Job		290
. 11.2	•		290
	11.20.1 Plate with a Hole in Tension		290
11.2	•		292
11.2	5		292
11.2			293
11.2			294
11.2			295
11.2			295
11.2	11 7 0 7		295
11.2	5		297
	9 Creating and Submitting an Analysis Job		298
11.3	· ·		299
11.3	1 Refining the Mesh		299
Append	x		303
A.1	Rotation of Coordinate System in Three Dimensions		303
A.2	Scalar Product Theorem		304
A.3	Taylor's Formula with Remainder and the Mean Value Theorem		304
A.4	Green's Theorem		305
A.5	Point Force (Source)		307
A.6	Static Condensation		308
A.7	Solution Methods		309
	Direct Solvers		310
	Iterative Solvers		310
<b>~</b> ~	Conditioning		311
	rences		312
Prob	lem		312
Index			313