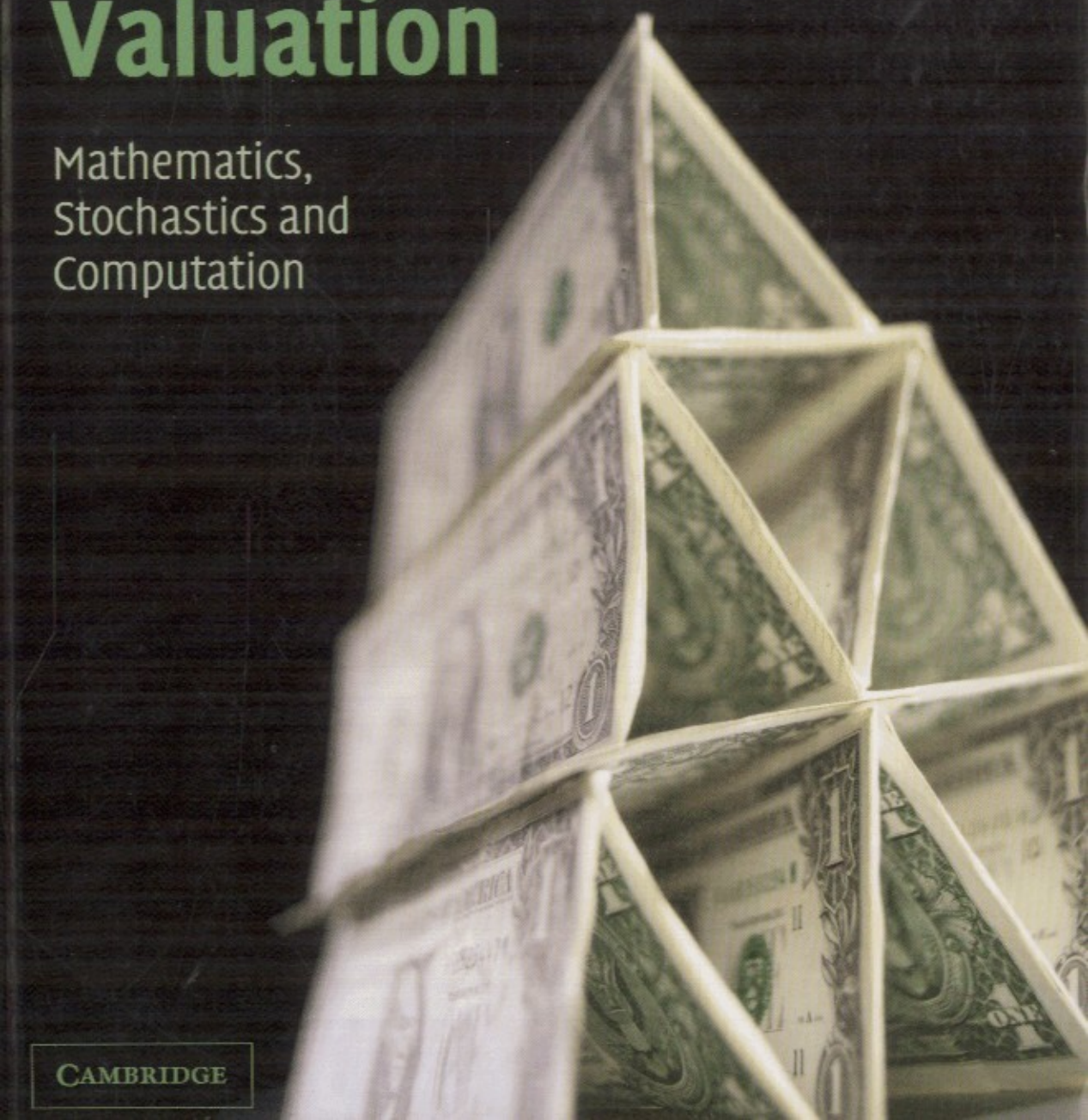
Desmond J. Higham

An Introduction to Financial Option Valuation

Mathematics, Stochastics and Computation



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