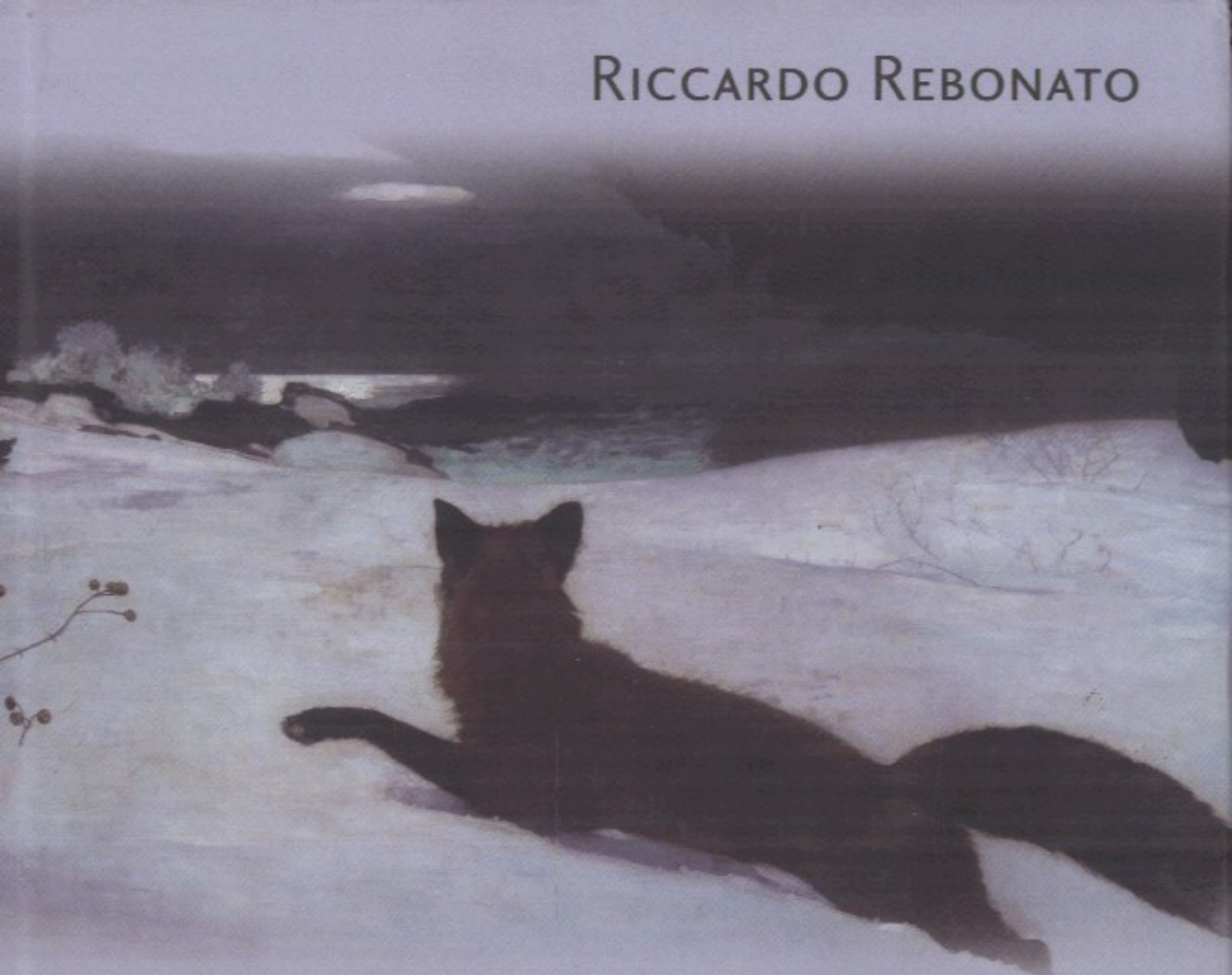


RICCARDO REBONATO



Volatility and Correlation

The Perfect Hedger
and the Fox

Second Edition

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