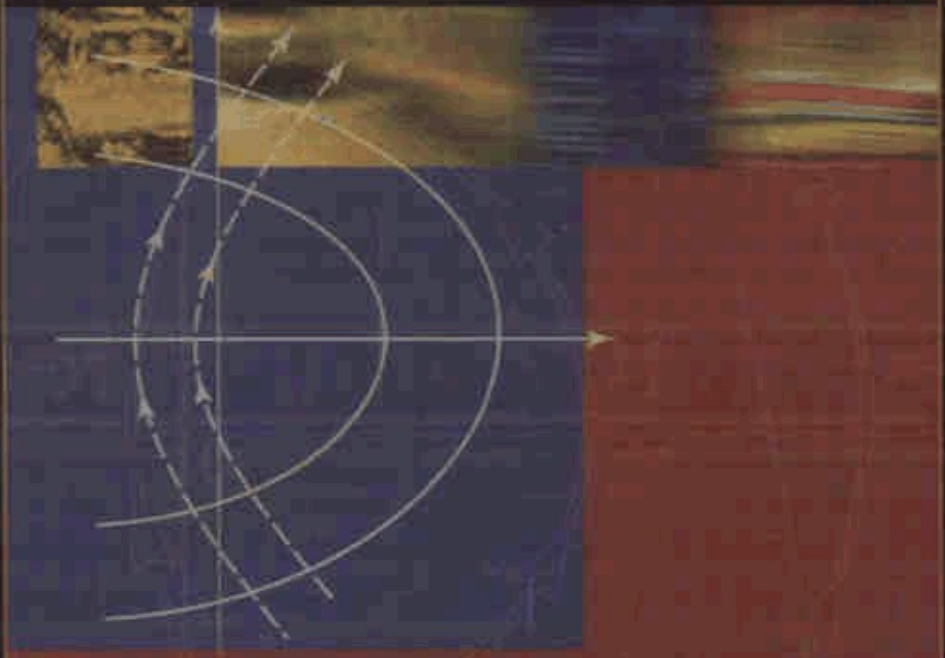
INTERNATIONAL EDITION

3rd Edition

Complex Variables with Applications



A. David Wunsch

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