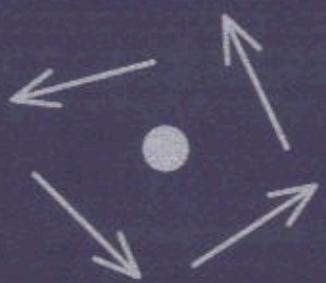
Igor Herbut

A Modern Approach to Critical Phenomena



CAMBRIDGE

Contents

Preface *page* ix

1 Introduction 1
1.1 Phase transitions and order parameters 1
1.2 Models: Ising, XY, Heisenberg 3
1.3 Universality and critical exponents 10
1.4 Scaling of free energy 12
1.5 Correlations and hyperscaling 19

2 Ginzburg–Landau–Wilson theory 23
2.1 Partition function for interacting bosons 23
2.2 Bose–Einstein condensation 27
2.3 Hartree approximation 30
2.4 Landau's mean-field theory 34
2.5 Upper critical dimension 40

3 Renormalization group 43
3.1 Idea 43
3.2 Momentum-shell transformation 46
3.3 ϵ-expansion 53
3.4 Dangerously irrelevant coupling 58
3.5 Corrections to scaling 59
3.6 Field-theoretic perspective 61
3.7 Computation of anomalous dimension 65
3.8 Summary 69

4 Superconducting transition 77
4.1 Meissner effect 77
4.2 Fluctuation-induced first-order transition 81
4.3 Type-II superconductors near four dimensions 85
4.4 Anomalous dimension for the gauge field 92
4.5 Width of the critical region 93

5 Near lower critical dimension 97
5.1 Goldstone modes 97
5.2 Mermin–Wagner–Hohenberg theorem 99
5.3 Non-linear σ-model 102
5.4 Low-temperature expansion 104
5.5 Discussion 106

6 Kosterlitz–Thouless transition 115
6.1 Vortices and spin waves 115
6.2 Mean-field theory 118
6.3 Duality and the sine-Gordon theory 123
6.4 Renormalization of the sine-Gordon model 131
6.5 Universal jump of superfluid density 138
6.6 Heisenberg model 140

7 Duality in higher dimensions 147
7.1 Frozen lattice superconductor 147
7.2 Confinement of magnetic monopoles 152
7.3 Magnetic field correlations 154
7.4 Compact electrodynamics 156

8 Quantum phase transitions 165
8.1 Dynamical critical exponent 165
8.2 Quantum critical point in Φ^4-theory 168
8.3 Bose–Hubbard model 174
8.4 Quantum fluctuations and the superfluid density 181
8.5 Universal conductivity in two dimensions 185

Appendix A Hubbard–Stratonovich transformation 195
Appendix B Linked-cluster theorem 197
Appendix C Gauge fixing for long-range order 199

Select bibliography 203
Index 209