

Guochang Xu

GPS

Theory, Algorithms
and Applications



2nd Edition



Springer

Contents

1	Introduction	1
1.1	A Key Note of GPS	2
1.2	A Brief Message About GLONASS	3
1.3	Basic Information of Galileo	4
1.4	A Combined Global Navigation Satellite System	5
2	Coordinate and Time Systems	7
2.1	Geocentric Earth-Fixed Coordinate Systems	7
2.2	Coordinate System Transformations	10
2.3	Local Coordinate System	11
2.4	Earth-Centred Inertial Coordinate System	13
2.5	Geocentric Ecliptic Inertial Coordinate System	17
2.6	Time Systems	17
3	Satellite Orbits	21
3.1	Keplerian Motion	21
3.1.1	Satellite Motion in the Orbital Plane	24
3.1.2	Keplerian Equation	27
3.1.3	State Vector of the Satellite	29
3.2	Disturbed Satellite Motion	31
3.3	GPS Broadcast Ephemerides	32
3.4	IGS Precise Ephemerides	34
3.5	GLONASS Ephemerides	35
4	GPS Observables	37
4.1	Code Pseudoranges	37
4.2	Carrier Phases	39
4.3	Doppler Measurements	41
5	Physical Influences of GPS Surveying	43
5.1	Ionospheric Effects	43
5.1.1	Code Delay and Phase Advance	43
5.1.2	Elimination of the Ionospheric Effects	45
5.1.3	Ionospheric Models	48
5.1.4	Mapping Functions	51

5.2	Tropospheric Effects	55
5.2.1	Tropospheric Models	56
5.2.2	Mapping Functions and Parameterisation	59
5.3	Relativistic Effects	62
5.3.1	Special Relativity and General Relativity	62
5.3.2	Relativistic Effects on GPS	64
5.4	Earth Tide and Ocean Loading Tide Corrections	67
5.4.1	Earth Tide Displacements of the GPS Station	67
5.4.2	Simplified Model of the Earth Tide Displacements	68
5.4.3	Numerical Examples of the Earth Tide Effects	70
5.4.4	Ocean Loading Tide Displacement	72
5.4.5	Computation of the Ocean Loading Tide Displacement	75
5.4.6	Numerical Examples of Loading Tide Effects	75
5.5	Clock Errors	76
5.6	Multipath Effects	78
5.6.1	GPS-Altimetry, Signals Reflected from the Earth-Surface	79
5.6.2	Reflecting Point Positioning	80
5.6.3	Image Point and Reflecting Surface Determination	81
5.7	Anti-Spoofing and Selective Availability Effects	82
5.8	Antenna Phase Centre Offset and Variation	82
5.9	Instrumental Biases	85
6	GPS Observation Equations and Equivalence Properties	87
6.1	General Mathematical Models of GPS Observations	87
6.2	Linearisation of the Observational Model	89
6.3	Partial Derivatives of Observational Function	90
6.4	Linear Transformation and Covariance Propagation	94
6.5	Data Combinations	95
6.5.1	Ionosphere-Free Combinations	97
6.5.2	Geometry-Free Combinations	98
6.5.3	Standard Phase-Code Combination	100
6.5.4	Ionospheric Residuals	101
6.5.5	Differential Doppler and Doppler Integration	102
6.6	Data Differentiations	104
6.6.1	Single Differences	105
6.6.2	Double Differences	107
6.6.3	Triple Differences	110
6.7	Equivalence of the Uncombined and Combining Algorithms	111
6.7.1	Uncombined GPS Data Processing Algorithms	112
6.7.2	Combining Algorithms of GPS Data Processing	114
6.7.3	Secondary GPS Data Processing Algorithms	119
6.7.4	Summary	122
6.8	Equivalence of Undifferenced and Differencing Algorithms	122
6.8.1	Introduction	122
6.8.2	Formation of Equivalent Observation Equations	123
6.8.3	Equivalent Equations of Single Differences	125

6.8.4	Equivalent Equations of Double Differences	128
6.8.5	Equivalent Equations of Triple Differences	130
6.8.6	Method of Dealing with the Reference Parameters	130
6.8.7	Summary of the Unified Equivalent Algorithm	131
7	Adjustment and Filtering Methods	133
7.1	Introduction	133
7.2	Least Squares Adjustment	133
7.2.1	Least Squares Adjustment with Sequential Observation Groups	135
7.3	Sequential Least Squares Adjustment	137
7.4	Conditional Least Squares Adjustment	138
7.4.1	Sequential Application of Conditional Least Squares Adjustment	140
7.5	Block-Wise Least Squares Adjustment	141
7.5.1	Sequential Solution of Block-Wise Least Squares Adjustment	143
7.5.2	Block-Wise Least Squares for Code-Phase Combination	145
7.6	Equivalently Eliminated Observation Equation System	146
7.6.1	Diagonalised Normal Equation and the Equivalent Observation Equation	148
7.7	Kalman Filter	150
7.7.1	Classic Kalman Filter	150
7.7.2	Kalman Filter – A General Form of Sequential Least Squares Adjustment	151
7.7.3	Robust Kalman Filter	152
7.7.4	Adaptively Robust Kalman Filtering	155
7.8	A Priori Constrained Least Squares Adjustment	159
7.8.1	A Priori Parameter Constraints	159
7.8.2	A Priori Datum	160
7.8.3	Quasi-Stable Datum	161
7.9	Summary	163
8	Cycle Slip Detection and Ambiguity Resolution	167
8.1	Cycle Slip Detection	167
8.2	Method of Dealing with Cycle Slips	168
8.3	A General Criterion of Integer Ambiguity Search	169
8.3.1	Introduction	169
8.3.2	Summary of Conditional Least Squares Adjustment	170
8.3.3	Float Solution	171
8.3.4	Integer Ambiguity Search in Ambiguity Domain	172
8.3.5	Integer Ambiguity Search in Coordinate and Ambiguity Domains	174
8.3.6	Properties of the General Criterion	175
8.3.7	An Equivalent Ambiguity Search Criterion and its Properties	176
8.3.8	Numerical Examples of the Equivalent Criterion	178
8.3.9	Conclusions and Comments	181
8.4	Ambiguity Function	182
8.4.1	Maximum Property of Ambiguity Function	183

9 Parameterisation and Algorithms of GPS Data Processing	187
9.1 Parameterisation of the GPS Observation Model	187
9.1.1 Evidence of the Parameterisation Problem of the Undifferenced Observation Model	187
9.1.2 A Method of Uncorrelated Bias Parameterisation	189
9.1.3 Geometry-Free Illustration	195
9.1.4 Correlation Analysis in the Case of Phase-Code Combinations	195
9.1.5 Conclusions and Comments	197
9.2 Equivalence of the GPS Data Processing Algorithms	198
9.2.1 Equivalence Theorem of GPS Data Processing Algorithms	198
9.2.2 Optimal Baseline Network Forming and Data Condition	200
9.2.3 Algorithms Using Secondary GPS Observables	201
9.3 Non-Equivalent Algorithms	203
9.4 Standard Algorithms of GPS Data Processing	203
9.4.1 Preparation of GPS Data Processing	203
9.4.2 Single Point Positioning	204
9.4.3 Standard Un-Differential GPS Data Processing	209
9.4.4 Equivalent Method of GPS Data Processing	211
9.4.5 Relative Positioning	212
9.4.6 Velocity Determination	212
9.4.7 Kalman Filtering Using Velocity Information	215
9.5 Accuracy of the Observational Geometry	217
10 Applications of GPS Theory and Algorithms	219
10.1 Software Development	219
10.1.1 Functional Library	219
10.1.2 Data Platform	223
10.1.3 A Data Processing Core	225
10.2 Concept of Precise Kinematic Positioning and Flight-State Monitoring	226
10.2.1 Introduction	226
10.2.2 Concept of Precise Kinematic Positioning	229
10.2.3 Concept of Flight-State Monitoring	233
10.2.4 Results, Precision Estimation and Comparisons	235
10.2.5 Conclusions	240
11 Perturbed Orbit and its Determination	243
11.1 Perturbed Equation of Satellite Motion	243
11.1.1 Lagrangian Perturbed Equation of Satellite Motion	244
11.1.2 Gaussian Perturbed Equation of Satellite Motion	246
11.2 Perturbation Forces of Satellite Motion	249
11.2.1 Perturbation of the Earth's Gravitational Field	249
11.2.2 Perturbation of the Sun and the Moon as well as Planets	254
11.2.3 Earth Tide and Ocean Tide Perturbations	255
11.2.4 Solar Radiation Pressure	258
11.2.5 Atmospheric Drag	262
11.2.6 Additional Perturbations	265

11.2.7 Order Estimations of Perturbations	267
11.2.8 Ephemerides of the Moon, the Sun and Planets	267
11.3 Analysis Solution of the \bar{C}_{20} Perturbed Orbit	271
11.4 Orbit Correction	277
11.5 Principle of GPS Precise Orbit Determination	281
11.5.1 Algebra Solution of the Variation Equation	283
11.6 Numerical Integration and Interpolation Algorithms	284
11.6.1 Runge-Kutta Algorithms	284
11.6.2 Adams Algorithms	289
11.6.3 Cowell Algorithms	291
11.6.4 Mixed Algorithms and Discussions	293
11.6.5 Interpolation Algorithms	294
11.7 Orbit-Related Partial Derivatives	294
12 Discussions	305
12.1 Independent Parameterisation and A Priori Information	305
12.2 Equivalence of the GPS Data Processing Algorithms	307
Appendix 1	
IAU 1980 Theory of Nutation	309
Appendix 2	
Numerical Examples of the Diagonalisation of the Equations	311
References	317
Subject Index	337