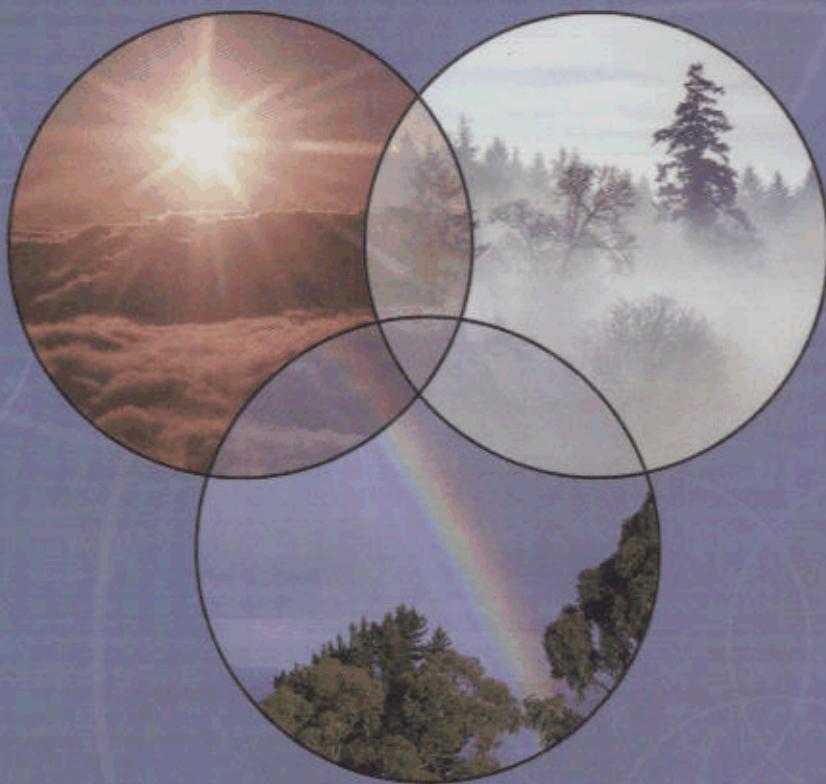


INTERNATIONAL GEOPHYSICS SERIES

STATISTICAL  
METHODS  
IN THE  
ATMOSPHERIC  
SCIENCES

*Second Edition*



DANIEL S. WILKS

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