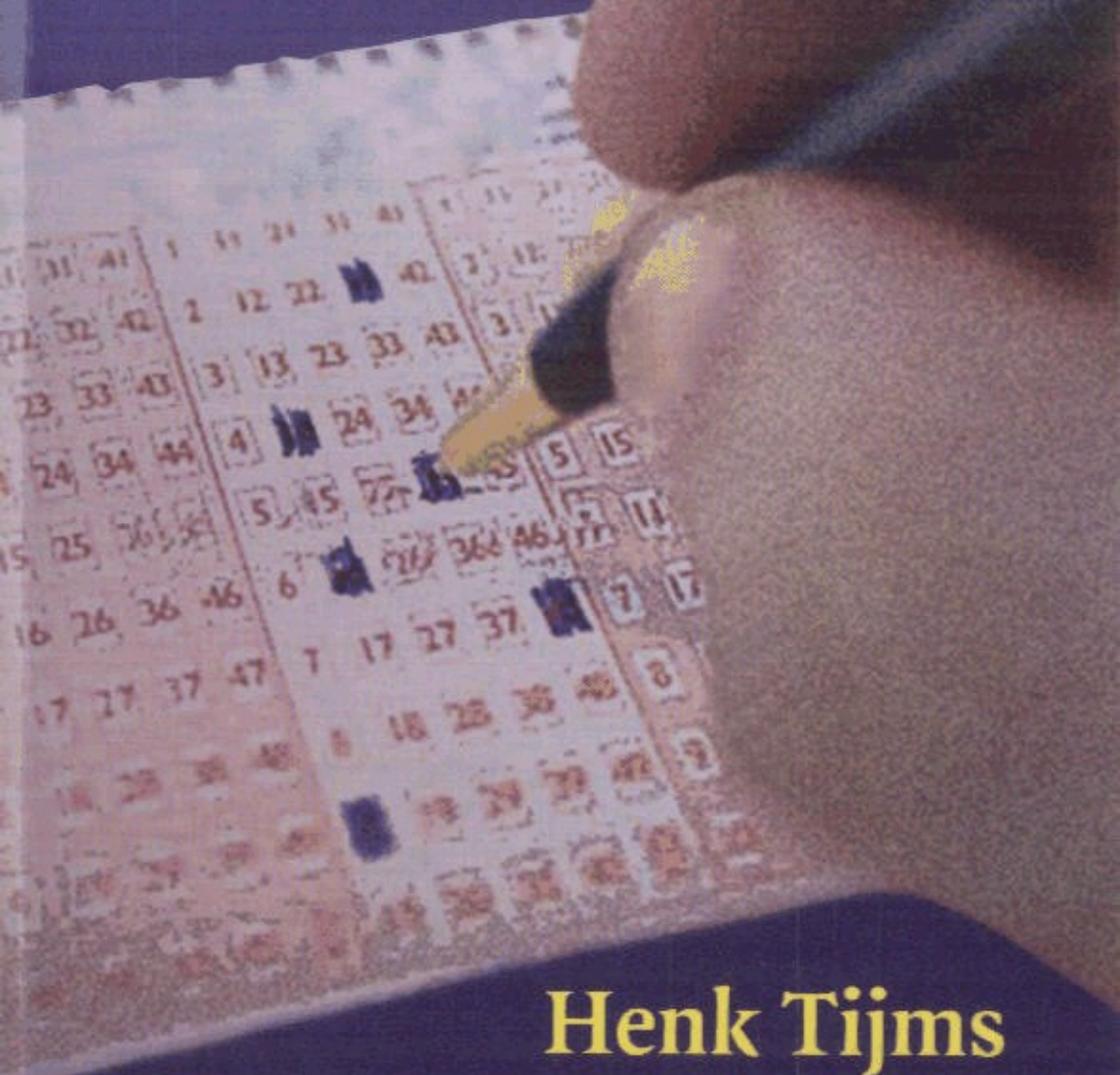


SECOND EDITION

Understanding Probability

Chance Rules in Everyday Life



Henk Tijms

CAMBRIDGE

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