Encyclopedia of Mathematics and Its Applications 113

STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS WITH LÉVY NOISE

S. Peszat and J. Zabczyk

CAMBRIDGE

Contents

Preface *page* ix

Part I Foundations 1

1 Why equations with Lévy noise? 3
- 1.1 Discrete-time dynamical systems 3
- 1.2 Deterministic continuous-time systems 5
- 1.3 Stochastic continuous-time systems 6
- 1.4 Courrège's theorem 8
- 1.5 Itô's approach 9
- 1.6 Infinite-dimensional case 12

2 Analytic preliminaries 13
- 2.1 Notation 13
- 2.2 Sobolev and Hölder spaces 13
- 2.3 L^p- and C_ρ-spaces 15
- 2.4 Lipschitz functions and composition operators 16
- 2.5 Differential operators 17

3 Probabilistic preliminaries 20
- 3.1 Basic definitions 20
- 3.2 Kolmogorov existence theorem 22
- 3.3 Random elements in Banach spaces 23
- 3.4 Stochastic processes in Banach spaces 25
- 3.5 Gaussian measures on Hilbert spaces 28
- 3.6 Gaussian measures on topological spaces 30
- 3.7 Submartingales 31
- 3.8 Semimartingales 36
- 3.9 Burkholder–Davies–Gundy inequalities 37

4 Lévy processes 38
4.1 Basic properties 38
4.2 Two building blocks – Poisson and Wiener processes 40
4.3 Compound Poisson processes in a Hilbert space 45
4.4 Wiener processes in a Hilbert space 50
4.5 Lévy–Khinchin decomposition 52
4.6 Lévy–Khinchin formula 56
4.7 Laplace transforms of convolution semigroups 57
4.8 Expansion with respect to an orthonormal basis 62
4.9 Square integrable Lévy processes 65
4.10 Lévy processes on Banach spaces 72
5 Lévy semigroups 75
5.1 Basic properties 75
5.2 Generators 78
6 Poisson random measures 83
6.1 Introduction 83
6.2 Stochastic integral of deterministic fields 85
6.3 Application to construction of Lévy processes 87
6.4 Moment estimates in Banach spaces 90
7 Cylindrical processes and reproducing kernels 91
7.1 Reproducing kernel Hilbert space 91
7.2 Cylindrical Poisson processes 100
7.3 Compensated Poisson measure as a martingale 105
8 Stochastic integration 107
8.1 Operator-valued angle bracket process 107
8.2 Construction of the stochastic integral 111
8.3 Space of integrands 114
8.4 Local properties of stochastic integrals 117
8.5 Stochastic Fubini theorem 118
8.6 Stochastic integral with respect to a Lévy process 121
8.7 Integration with respect to a Poisson random measure 125
8.8 L^p-theory for vector-valued integrands 130

Part II Existence and Regularity 137

9 General existence and uniqueness results 139
9.1 Deterministic linear equations 139
9.2 Mild solutions 141
9.3 Equivalence of weak and mild solutions 148
9.4 Linear equations 155

	9.5	Existence of weak solutions	164
	9.6	Markov property	167
	9.7	Equations with general Lévy processes	170
	9.8	Generators and a martingale problem	174
10	**Equations with non-Lipschitz coefficients**		179
	10.1	Dissipative mappings	179
	10.2	Existence theorem	183
	10.3	Reaction–diffusion equation	187
11	**Factorization and regularity**		190
	11.1	Finite-dimensional case	190
	11.2	Infinite-dimensional case	193
	11.3	Applications to time continuity	197
	11.4	The case of an arbitrary martingale	199
12	**Stochastic parabolic problems**		201
	12.1	Introduction	201
	12.2	Space–time continuity in the Wiener case	208
	12.3	The jump case	214
	12.4	Stochastic heat equation	219
	12.5	Equations with fractional Laplacian and stable noise	223
13	**Wave and delay equations**		225
	13.1	Stochastic wave equation on $[0, 1]$	225
	13.2	Stochastic wave equation on $\mathbb{R}^d$ driven by impulsive noise	230
	13.3	Stochastic delay equations	238
14	**Equations driven by a spatially homogeneous noise**		240
	14.1	Tempered distributions	240
	14.2	Lévy processes in $S'(\mathbb{R}^d)$	241
	14.3	RKHS of a square integrable Lévy process in $S'(\mathbb{R}^d)$	242
	14.4	Spatially homogeneous Lévy processes	246
	14.5	Examples	248
	14.6	RKHS of a homogeneous noise	253
	14.7	Stochastic equations on $\mathbb{R}^d$	255
	14.8	Stochastic heat equation	256
	14.9	Space–time regularity in the Wiener case	261
	14.10	Stochastic wave equation	267
15	**Equations with noise on the boundary**		272
	15.1	Introduction	272
	15.2	Weak and mild solutions	275
	15.3	Analytical preliminaries	277
	15.4	L^2 case	279
	15.5	Poisson perturbation	282

Part III Applications 285

16 Invariant measures 287
16.1 Basic definitions 287
16.2 Existence results 289
16.3 Invariant measures for the reaction–diffusion equation 297

17 Lattice systems 299
17.1 Introduction 299
17.2 Global interactions 300
17.3 Regular case 303
17.4 Non-Lipschitz case 305
17.5 Kolmogorov's formula 306
17.6 Gibbs measures 307

18 Stochastic Burgers equation 312
18.1 Burgers system 312
18.2 Uniqueness and local existence of solutions 314
18.3 Stochastic Burgers equation with additive noise 317

19 Environmental pollution model 322
19.1 Model 322

20 Bond market models 324
20.1 Forward curves and the HJM postulate 324
20.2 HJM condition 327
20.3 HJMM equation 332
20.4 Linear volatility 342
20.5 BGM equation 347
20.6 Consistency problem 350

Appendix A Operators on Hilbert spaces 355
Appendix B C_0-semigroups 365
Appendix C Regularization of Markov processes 388
Appendix D Itô formulae 391
Appendix E Lévy–Khinchin formula on $[0, +\infty)$ 394
Appendix F Proof of Lemma 4.24 396
List of symbols 399
References 403
Index 415