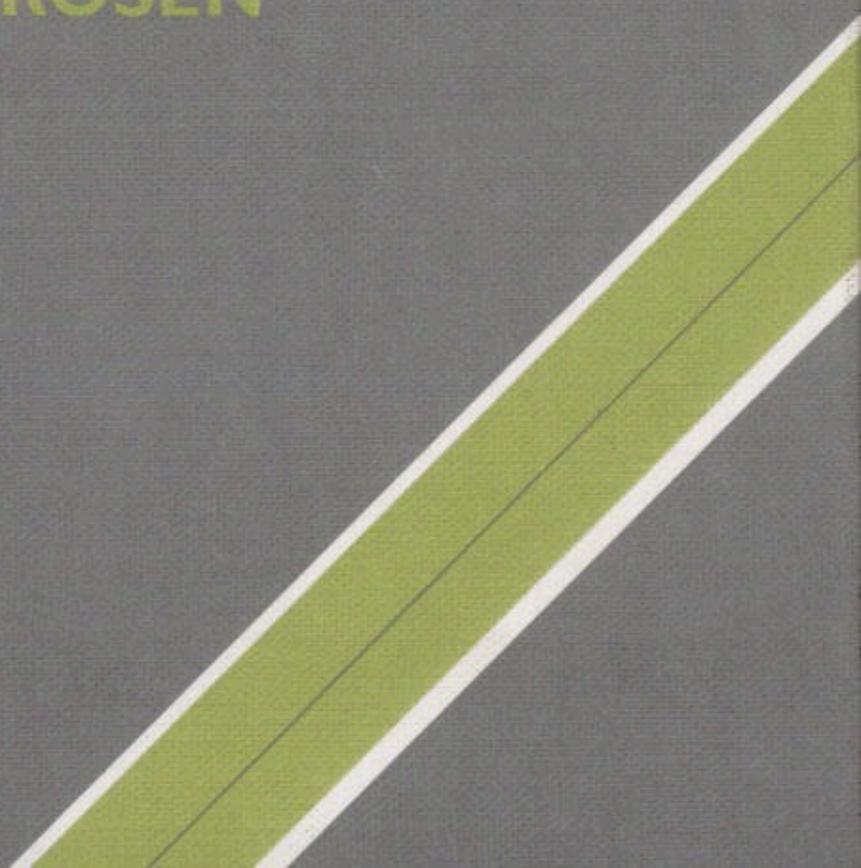


Cambridge studies in advanced mathematics 100

# Markov Processes, Gaussian Processes, and Local Times

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