

Graduate Texts in Mathematics

Palle E.T. Jorgensen

Analysis and Probability

Wavelets, Signals,
Fractals



Springer

Contents

Preface	vii
Getting started	xv
An apology	xv
Glossary: <i>function, random variable, signal, state, sequence (incl. vector-valued), random walk, time-series, measurement, nested subspaces, refinement, multiresolution, scales of visual resolutions, operator, process, black box, observable (if selfadjoint), Fourier dual pair, generating function, time/frequency, P/Q, convolution, filter, smearing, decomposition (e.g., Fourier coefficients in a Fourier expansion), analysis, frequency components, integrate (e.g., inverse Fourier transform), reconstruct, synthesis, superposition, subspace, resolution, (signals in a) frequency band, Cuntz relations, perfect reconstruction from subbands, subband decomposition, inner product, correlation, transition probability, probability of transition from one state to another, $f_{\text{out}} = T f_{\text{in}}$, input/output, transformation of states, fractal, conditional expectation, martingale, data mining (A translation guide!)</i>	xvii
Multiresolutions	xxvi
Prerequisites and cross-audience	xxvii
Aim and scope	xxviii
Self-similarity	xxix
New issues, new tools	xxx
List of names and discoveries	xxx
General theory	xxxiv
A word about the graphics and the illustrations	xxxiv
Special features of the book	xxxv
Exercises: Overview	xxxvi
Figures. Read Me!	xxxix

Acknowledgments	xliii
1 Introduction: Measures on path space	1
Prerequisites	1
Prelude	1
1.1 Wavelets	2
1.2 Path space	6
1.3 Multiresolutions	9
1.4 Sampling	17
1.5 A convergence theorem for infinite products	18
1.6 A brief outline	21
1.7 From wavelets to fractals	22
Exercises	27
History	33
References and remarks	35
2 Transition probabilities: Random walk	39
Prerequisites	39
Prelude	39
2.1 Standing assumptions	40
2.2 An example	41
2.3 Some definitions: The Ruelle operator, harmonic functions, cocycles	43
2.4 Existence of the measures P_x	43
2.5 Kolmogorov's consistency condition	46
2.6 The probability space Ω	47
2.7 A boundary representation for harmonic functions	48
2.8 Invariant measures	52
Exercises	54
References and remarks	57
3 $\mathbb{N}_0$ vs. $\mathbb{Z}$	59
Prerequisites	59
Prelude	59
3.1 Terminology	60
3.2 The unit interval	62
3.3 A sufficient condition for $P_x(\mathbb{Z}) = 1$	64
Exercises	66
References and remarks	67
4 A case study: Duality for Cantor sets	69
Prerequisites	69
Prelude	69

4.1	Affine iterated function systems: The general case	70
4.2	The quarter Cantor set: The example $W(x) = \cos^2(2\pi x)$	72
4.3	The conjugate Cantor set, and a special harmonic function	76
4.4	A sufficient condition for $P_x(\mathbb{N}_0) = 1$	78
	Conclusions	79
	Exercises	79
	References and remarks	80
5	Infinite products	83
	Prerequisites	83
	Prelude	83
5.1	Riesz products	84
5.2	Random products	84
5.3	The general case	85
5.4	A uniqueness theorem	86
5.5	Wavelets revisited	91
	Exercises	93
	References and remarks	97
6	The minimal eigenfunction	99
	Prerequisites	99
	Prelude	99
6.1	A general construction of $h_{\min}$	100
6.2	A closed expression for $h_{\min}$	102
	Exercises	106
	References and remarks	107
7	Generalizations and applications	109
	Prerequisites	109
	Prelude	109
7.1	Translations and the spectral theorem	110
7.2	Multiwavelets and generalized multiresolution analysis (GMRA)	114
7.3	Operator-coefficients	114
7.4	Operator-valued measures	115
7.5	Wavelet packets	122
7.6	Representations of the Cuntz algebra $\mathcal{O}_2$	131
7.7	Representations of the algebra of the canonical anticommutation relations (CARs)	138
	Exercises	141
	References and remarks	151

8	Pyramids and operators	157
	Prerequisites	157
	Prelude	157
8.1	Why pyramids	158
8.2	Dyadic wavelet packets	159
8.3	Measures and decompositions	166
8.4	Multiresolutions and tensor products	168
	Exercises	173
	References and remarks	176
9	Pairs of representations of the Cuntz algebras $\mathcal{O}_n$, and their application to multiresolutions	179
	Prerequisites	179
	Prelude	179
9.1	Factorization of unitary operators in Hilbert space	180
9.2	Generalized multiresolutions	181
9.3	Permutation of bases in Hilbert space	182
9.4	Tilings	185
9.5	Applications to wavelets	190
9.6	An application to fractals	194
9.7	Phase modulation	198
	Exercises	199
	References and remarks	204
	Appendices: Polyphase matrices and the operator algebra $\mathcal{O}_N$	205
	Prerequisites	205
	Prelude	205
	Appendix A: Signals and filters	206
	Appendix B: Hilbert space and systems of operators	210
	Appendix C: A tale of two Hilbert spaces	212
	Table C.1: Operations on two Hilbert spaces: The correspondence principle.	213
	Appendix D: Signal processing, matrices, and programming diagrams	218
	References and remarks: Systems theory	221
	Afterword	223
	Comments on signal/image processing terminology	223
	Introduction	223
	JPEG 2000 vs. GIF	225
	JPEG 2000	225
	GIF	226
	Grayscale	227

Quadrature-mirror filter	227
What is a <i>frame</i> ?	228
To the mathematics student	228
To an engineer	229
Alias (aliasing)	229
Engineering	229
Mathematics	229
Computational mathematics	230
Epigraphs	233
References	237
Symbols	251
Index	259