Lecture Notes in Mathematics

1905

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A Concise Course on Stochastic Partial Differential Equations

$$\boxed{\mathrm{d}X_t = [\Delta\Psi(X_t) + \Phi(X_t)]\,\mathrm{d}t + B(X_t)\,\mathrm{d}W_t}$$

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