

CHAPMAN & HALL/CRC APPLIED MATHEMATICS
AND NONLINEAR SCIENCE SERIES

Stochastic Partial Differential Equations

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Contents

Preface	iii
1 Preliminaries	1
1.1 Introduction	1
1.2 Some Examples	4
1.3 Brownian Motions and Martingales	6
1.4 Stochastic Integrals	8
1.5 Stochastic Differential Equations	12
1.6 Comments	14
2 Scalar Equations of First Order	15
2.1 Introduction	15
2.2 Generalized Itô's Formula	17
2.3 Linear Stochastic Equations	22
2.4 Quasilinear Equations	28
2.5 General Remarks	31
3 Stochastic Parabolic Equations	33
3.1 Introduction	33
3.2 Preliminaries	34
3.3 Solution of Random Heat Equation	43
3.4 Linear Equations with Additive Noise	49
3.5 Some Regularity Properties	54
3.6 Random Reaction-Diffusion Equations	65
3.7 Parabolic Equations with Gradient-Dependent Noise	74
4 Stochastic Parabolic Equations in the Whole Space	85
4.1 Introduction	85
4.2 Preliminaries	86
4.3 Linear and Similinear Equations	89
4.4 Feynman-Kac Formula	94
4.5 Positivity of Solutions	97
4.6 Correlation Functions of Solutions	102

5	Stochastic Hyperbolic Equations	109
5.1	Introduction	109
5.2	Preliminaries	110
5.3	Wave Equation with Additive Noise	110
5.4	Semilinear Wave Equations	118
5.5	Wave Equations in Unbounded Domain	123
5.6	Randomly Perturbed Hyperbolic Systems	127
6	Stochastic Evolution Equations in Hilbert Spaces	139
6.1	Introduction	139
6.2	Hilbert Space-Valued Martingales	140
6.3	Stochastic Integrals in Hilbert Spaces	145
6.4	Itô's Formula	152
6.5	Stochastic Evolution Equations	154
6.6	Mild Solutions	157
6.7	Strong Solutions	168
6.8	Stochastic Evolution Equations of the Second Order	185
7	Asymptotic Behavior of Solutions	195
7.1	Introduction	195
7.2	Itô's Formula and Lyapunov Functionals	196
7.3	Boundedness of Solutions	199
7.4	Stability of Null Solution	202
7.5	Invariant Measures	205
7.6	Small Random Perturbation Problems	213
7.7	Large Deviations Problems	218
8	Further Applications	223
8.1	Introduction	223
8.2	Stochastic Burgers and Related Equations	224
8.3	Random Schrödinger Equation	225
8.4	Nonlinear Stochastic Beam Equations	227
8.5	Stochastic Stability of Cahn-Hilliard Equation	230
8.6	Invariant Measures for Stochastic Navier-Stokes Equations	232
9	Diffusion Equations in Infinite Dimensions	237
9.1	Introduction	237
9.2	Diffusion Processes and Kolmogorov Equations	239
9.3	Gauss-Sobolev Spaces	245
9.4	Ornstein-Uhlenbeck Semigroup	250
9.5	Parabolic Equations and Related Elliptic Problems	257
9.6	Characteristic Functionals and Hopf Equations	266
	References	273

