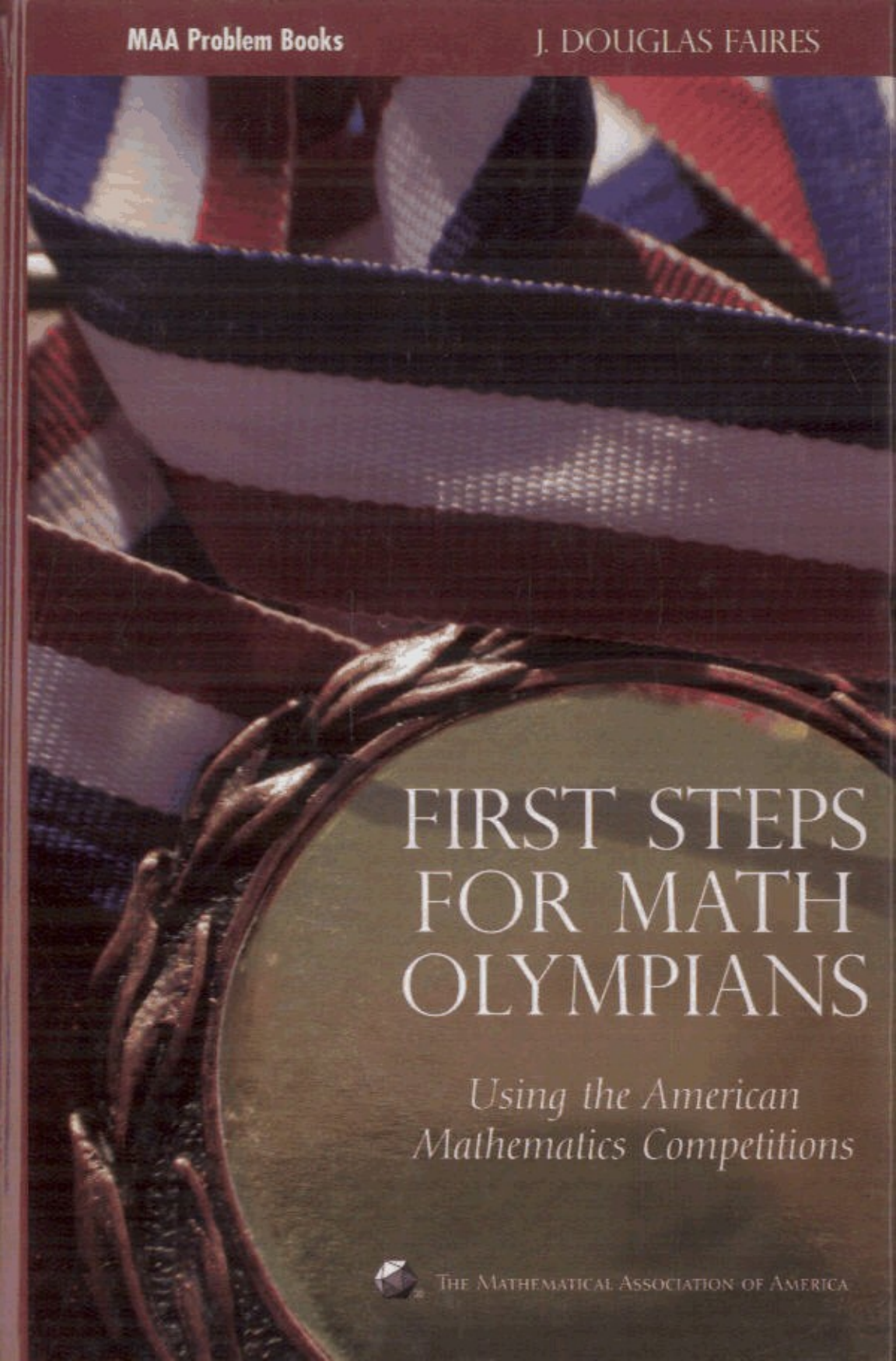


MAA Problem Books

J. DOUGLAS FAIRES



FIRST STEPS
FOR MATH
OLYMPIANS

*Using the American
Mathematics Competitions*



THE MATHEMATICAL ASSOCIATION OF AMERICA

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