

Control Perspectives on Numerical Algorithms and Matrix Problems



Amit Bhaya
Eugenius Kaszkurewicz

Advances in Design and Control

siam

Contents

List of Figures	xi
List of Tables	xvii
Preface	xix
1 Brief Review of Control and Stability Theory	1
1.1 Control Theory Basics	1
1.1.1 Feedback control terminology	1
1.1.2 PID control for discrete-time systems	6
1.2 Optimal Control Theory	8
1.3 Linear Systems, Transfer Functions, Realization Theory	10
1.4 Basics of Stability of Dynamical Systems	15
1.5 Variable Structure Control Systems	27
1.6 Gradient Dynamical Systems	31
1.6.1 Nonsmooth GDSs: Persidskii-type results	35
1.7 Notes and References	38
2 Algorithms as Dynamical Systems with Feedback	41
2.1 Continuous-Time Dynamical Systems that Find Zeros	42
2.2 Iterative Zero Finding Algorithms as Discrete-Time Dynamical Systems	56
2.3 Iterative Methods for Linear Systems as Feedback Control Systems . .	70
2.3.1 CLF/LOC derivation of minimal residual and Krylov subspace methods	75
2.3.2 The conjugate gradient method derived from a proportional-derivative controller	77
2.3.3 Continuous algorithms for finding optima and the continuous conjugate gradient algorithm	85
2.4 Notes and References	90
3 Optimal Control and Variable Structure Design of Iterative Methods	93
3.1 Optimally Controlled Zero Finding Methods	94
3.1.1 An optimal control-based Newton-type method	94
3.2 Variable Structure Zero Finding Methods	96

3.2.1	A variable structure Newton method to find zeros of a polynomial function	99
3.2.2	The spurt method	107
3.3	Optimal Control Approach to Unconstrained Optimization Problems .	109
3.4	Differential Dynamic Programming Applied to Unconstrained Minimization Problems	118
3.5	Notes and References	124
4	Neural-Gradient Dynamical Systems for Linear and Quadratic Programming Problems	127
4.1	GDSs, Neural Networks, and Iterative Methods	128
4.2	GDSs that Solve Linear Systems of Equations	140
4.3	GDSs that Solve Convex Programming Problems	146
4.3.1	Stability analysis of a class of discontinuous GDSs	150
4.4	GDSs that Solve Linear Programming Problems	154
4.4.1	GDSs as linear programming solvers	159
4.5	Quadratic Programming and Support Vector Machines	167
4.5.1	ν -support vector classifiers for nonlinear separation via GDSs	170
4.6	Further Applications: Least Squares Support Vector Machines, K-Winners-Take-All Problem	173
4.6.1	A least squares support vector machine implemented by a GDS	173
4.6.2	A GDS that solves the k-winners-take-all problem	174
4.7	Notes and References	177
5	Control Tools in the Numerical Solution of Ordinary Differential Equations and in Matrix Problems	179
5.1	Stepsize Control for ODEs	179
5.1.1	Stepsize control as a linear feedback system	182
5.1.2	Optimal stepsize control for ODEs	185
5.2	A Feedback Control Perspective on the Shooting Method for ODEs . .	191
5.2.1	A state space representation of the shooting method . . .	192
5.2.2	Error dynamics of the iterative shooting scheme	195
5.3	A Decentralized Control Perspective on Diagonal Preconditioning . .	199
5.3.1	Perfect diagonal preconditioning	203
5.3.2	LQ perspective on optimal diagonal preconditioners . . .	208
5.4	Characterization of Matrix D-Stability Using Positive Realness of a Feedback System	210
5.4.1	A feedback control approach to the D-stability problem via strictly positive real functions	213
5.4.2	D-stability conditions for matrices of orders 2 and 3 . . .	216
5.5	Finding Zeros of Two Polynomial Equations in Two Variables via Controllability and Observability	218
5.6	Notes and References	224

Contents	ix
6 Epilogue	227
Bibliography	233
Index	255