



ENGINEERING APPLICATIONS OF DYNAMICS

Dean C. Karnopp • Donald L. Margolis

CONTENTS

Preface xi

1 Newton's Laws for Particles and Rigid Bodies 1

- 1.1 Newton's Second Law, 2
- 1.2 Coordinate Frames and Velocity and Acceleration Diagrams, 3
 - Rectangular Coordinates, 3
 - Polar Coordinates, 4
 - Coordinate Choice and Degrees-of-Freedom, 8
- 1.3 Free-Body and Force Diagrams, 9
- 1.4 Transferring Velocity and Acceleration Components, 11
- 1.5 Transferring Motion Components of Rigid Bodies and Generating Kinematic Constraints, 16
 - Kinematic Constraints, 18
- 1.6 Review of Center of Mass, Linear Momentum, and Angular Momentum for Rigid Bodies, 21
- 1.7 Newton's Law Applied to Rigid Bodies, 25
- Reference, 27
- Problems, 27

2 Equations of Motion in Second- and First-Order Form 41

- 2.1 Deriving Equations of Motion for Systems of Particles, 41

- 2.2 Deriving Equations of Motion When Rigid Bodies Are Part of the System, 46
- 2.3 Forms of Equations and Their Computational Solution, 55
 - First-Order State Equations, 56
 - Explicit Form, 56
 - Fundamentals of Computer-Developed Time-Step Simulation, 57
 - Implicit Form, 57
 - Differential Algebraic Form, 58
- 2.4 Reducing Sets of Second-Order Differential Equations to First-Order Form, 58
- 2.5 Matrix Forms for Linearized Equations, 64
 - Quarter-Car Model for Vibration Analysis, 64
 - Half-Car Model for Vibration Analysis and Control, 68
 - Linearization of the Inverted Pendulum, 71
- 2.6 Summary, 73
- References, 74
- Problems, 74

3 Computer Solution of Equations of Motion 92

- 3.1 Time-Step Simulation of Nonlinear Equations of Motion, 92
- 3.2 Linear System Response, 107
 - Eigenvalues and Their Relationship to System Stability, 107
 - Transfer Functions, 110
 - Frequency Response, 114
- References, 118
- Problems, 119

4 Energy and Lagrange Equation Methods 135

- 4.1 Kinetic and Potential Energy, 136
- 4.2 Using Conservation of Energy to Derive Equations of Motion, 139
- 4.3 Equations of Motion from Lagrange's Equations, 141
 - Generalized Coordinates, 141
 - Lagrange's Equations, 144
 - Generalized Forces, 146
 - Imposed Motion, 148
- 4.4 Interpretation of Lagrange's Equations, 150
- 4.5 Nonlinear Kinematics and Lagrange's Equations, 152

- Approximate Method for Satisfying Constraints, 156
- 4.6 First-Order Forms for Lagrange's Equations, 158
 - Example System, 160
 - Comments Regarding the Use of p and q Variables in Simulation, 161
- 4.7 Nonholonomic Systems, 162
- 4.8 Summary, 162
- References, 163
- Problems, 163

5 Newton's Laws in a Body-Fixed Frame: Application to Vehicle Dynamics 180

- 5.1 The Dynamics of a Shopping Cart, 181
 - Inertial Coordinate System, 181
 - Body-Fixed Coordinate System, 187
 - Connection between Inertial and Body-Fixed Frames, 189
- 5.2 Analysis of a Simple Car Model, 190
- 5.3 Vehicle Stability, 193
- 5.4 Stability, Critical Speed, Understeer, and Oversteer, 196
- 5.5 Steering Transfer Functions, 197
 - Yaw Rate and Lateral Acceleration Gains, 200
 - Special Case of the Neutral Steering Vehicle, 200
- 5.6 Steady Cornering, 202
 - Description of Steady Turns, 202
 - Significance of the Understeering Coefficient, 204
 - Acceleration and Yaw Rate Gain Behavior, 205
- 5.7 Summary, 209
- References, 209
- Problems, 209

6 Mechanical Systems under Active Control 219

- 6.1 Basic Concepts, 220
 - Characteristic Equation, 221
 - Transfer Functions, 221
 - State-Variable Feedback, 222
- 6.2 State Variables and Active Control, 222
 - Compromises in Passive Vibration Isolation, 224
 - Active Control in Vibration Isolation, 226

- Optimized Active Vibration Isolator, 228
- 6.3 Steering Control of Banking Vehicles, 231
 - Development of the Mathematical Model, 232
 - Derivation of the Dynamic Equations, 234
 - Stability of the Lean Angle, 237
 - Steering Control of the Lean Angle, 238
 - Counter Steering or Reverse Action, 240
- 6.4 Active Control of Vehicle Dynamics, 242
 - Stability and Control, 243
 - From ABS to VDC, 244
 - Model Reference Control, 246
 - Active Steering Systems, 248
 - Stability Augmentation Using Front, Rear, or All-Wheel Steering, 249
 - Feedback Model Following Active Steering Control, 249
 - Sliding Mode Control, 251
 - Active Steering Applied to the Bicycle Model of an Automobile, 254
 - Active Steering Yaw Rate Controller, 256
 - Limitations of Active Stability Enhancement, 260
- 6.5 Summary, 261
- References, 261
- Problems, 262

7 Rigid-Body Motion in Three Dimensions 271

- 7.1 General Equations of Motion, 272
- 7.2 Use of a Body-Fixed Coordinate Frame, 275
 - Euler's Equations, 276
 - Spin Stabilization of Satellites, 277
- 7.3 Use of an Inertial Coordinate Frame, 281
 - Euler's Angles, 283
 - Kinetic Energy, 285
 - Steady Precession of Gyroscopes, 285
 - Dynamics of Gyroscopes, 287
- 7.4 Summary, 292
- References, 293
- Problems, 293

8 Vibration of Multiple-Degree-of-Freedom Systems 305

- 8.1 Natural Frequency and Resonance of a Single-Degree-of-Freedom Oscillator, 306
 - Free Response, 306
 - Forced Response, 309
 - Comparison of Two Suspension Geometries, 309
- 8.2 Two-Degree-of-Freedom Systems, 314
 - Free, Undamped Response, 315
 - Forced Response of Two-Degree-of-Freedom Systems, 326
- 8.3 Tuned Vibration Absorbers, 328
 - Some Configurations for TVAs, 332
- 8.4 Summary, 339
- References, 339
- Problems, 340

9 Distributed System Vibrations 350

- 9.1 Stress Waves in a Rod, 350
 - Free Response: Separation of Variables, 354
 - Forced Response, 357
 - Orthogonality of Mode Functions, 359
 - Representation of Point Forces, 359
 - Rigid-Body Mode, 362
 - Back to the Forced Response, 363
- 9.2 Attaching the Distributed System to External Dynamic Components, 366
- 9.3 Tightly Stretched Cable, 372
 - Free Response: Separation of Variables, 375
 - Forced Response, 377
- 9.4 Bernoulli–Euler Beam, 379
 - Free Response: Separation of Variables, 382
 - Forced Response, 384
- 9.5 Summary, 388
- References, 389
- Problems, 389

Appendix A: Three-Dimensional Rigid-Body Motion in a Rotating Coordinate System 402

- References, 407

x **CONTENTS**

Appendix B: Moments of Inertia for Some Common Body Shapes 408

Appendix C: Parallel Axis Theorem 410

Index 413