

## APPLIED MANOVA AND DISCRIMINANT ANALYSIS

Second Edition

CARL J. HUBERTY STEPHEN OLEJNIK



## Contents

Lis	t of Fi	gures	xix
Lis	t of Ta	ables	xxi
Pre	eface t	o Second Edition	xxv
Ac	knowl	edgments	xxvii
Pre	eface t	o First Edition	xxix
No	tation		xxxi
I	INT	RODUCTION	1
1	Disc	riminant Analysis in Research	3
	1.1 1.2 1.3 1.4 1.5	A Little History, 3 Overview, 5 Descriptive Discriminant Analysis, 5 Predictive Discriminant Analysis, 7 Design in Discriminant Analysis, 9 1.5.1 Grouping Variables, 9 1.5.2 Response Variables, 9 Exercises, 13	
2	Prel	iminaries	15
	2.1 2.2 2.3 2.4 2.5	Introduction, 15 Research Context, 15 Data, Analysis Units, Variables, and Constructs, 16 Summarizing Data, 18 Matrix Operations, 21 2.5.1 SSCP Matrix, 22	

X CONTENTS

	2.6 2.7 2.8 2.9 2.10 2.11	2.5.2 Determinant, 23 2.5.3 Inverse, 24 2.5.4 Eigenanalysis, 25 Distance, 26 Linear Composite, 28 Probability, 28 Statistical Testing, 29 Judgment in Data Analysis, 30 Summary, 31 Further Reading, 31 Exercises, 32	
II	ONE	E-FACTOR MANOVA/DDA	33
3	Grou	p Separation	35
	3.1	Introduction, 35	
	3.2	Two-Group Analyses, 35	
		<ul><li>3.2.1 Univariate Analysis, 35</li><li>3.2.2 Multivariate Analysis, 39</li></ul>	
	3.3	Test for Covariance Matrix Equality, 41	
	3.4	Yao Test, 43	
	3.5	Multiple-Group Analyses—Single Factor, 44 3.5.1 Univariate Analysis, 44 3.5.2 Multivariate Analysis, 47	
	3.6	Computer Application, 52	
	3.7	Summary, 56	
		Exercises, 57	
4	Asses	ssing MANOVA Effects	61
	4.1	Introduction, 61	
	4.2	Strength of Association, 62	
		4.2.1 Univariate, 62 4.2.2 Multivariate, 62 4.2.3 Bias, 65	
	4.3	Computer Application I, 66	
	4.4	Group Contrasts, 67	
		<ul><li>4.4.1 Univariate, 67</li><li>4.4.2 Multivariate, 68</li></ul>	
	4.5	Computer Application II, 72	
	4.6	Covariance Matrix Heterogeneity, 74	
	4.7	Sample Size, 74	

CONTENTS	xi
CONTENTS	734

	4.8	Summary, 75 Technical Notes, 76	
		Exercises, 77	
5	Desci	ribing MANOVA Effects	81
	5.1	Introduction, 81	
	5.2	Omnibus Effects, 82	
		<ul><li>5.2.1 An Eigenanalysis, 82</li><li>5.2.2 Linear Discriminant Functions, 83</li></ul>	
	5.3	Computer Application I, 85	
	5.4	Standardized LDF Weights, 87	
	5.5	LDF Space Dimension, 88	
		5.5.1 Statistical Tests, 89	
		5.5.2 Proportion of Variance, 91 5.5.3 LDF Plots, 91	
	5.6	Computer Application II, 93	
	5.7	Computer Application III, 94	
		Contrast Effects, 96	
	5.9	Computer Application IV, 96	
	5.10	Summary, 98	
		Technical Note, 99	
		Further Reading, 100	
		Exercises, 100	
6	Delet	ing and Ordering Variables	103
	6.1	Introduction, 103	
	6.2	Variable Deletion, 103	
		6.2.1 Purposes of Deletion, 103	
		6.2.2 McCabe Analysis, 104	
	6.3	6.2.3 Computer Application, 105 Variable Ordering, 106	
	0.5	6.3.1 Meaning of Importance, 106	
		6.3.2 Computer Application I, 108	
		6.3.3 Variable Ranking, 110	
	6.4	Contrast Analyses, 110	
	6.5	Computer Application II, 111	
	6.6	Comments, 113	
		Further Reading, 114	
		Exercises, 115	
7	Repo	orting DDA Results	117
	7.1	Introduction, 117	

Example of Reporting DDA Results, 117

7.2

**Xİİ** CONTENTS

	7.3	Computer Package Information, 122	
	7.4	Reporting Terms, 123	
	7.5	MANOVA/DDA Applications, 124	
	7.6	Concerns, 124	
	7.7	Overview, 126	
		Further Reading, 127	
		Exercises, 127	
Ш		TORIAL MANOVA, MANCOVA, AND REPEATED ASURES	129
8	Fact	torial MANOVA	131
	8.1	Introduction, 131	
	8.2	Research Context, 131	
	8.3	Univariate Analysis, 134	
	8.4	Multivariate Analysis, 136	
		8.4.1 Omnibus Tests, 136	
		8.4.2 Distribution Assumptions, 138	
	8.5	Computer Application I, 139	
	8.6	Computer Application II, 146	
	8.7	Nonorthogonal Design, 150	
	8.8	Outcome Variable Ordering and Deletion, 151	
	8.9	Summary, 152	
		Technical Notes, 152	
		Exercises, 159	
9	Ana	lysis of Covariance	163
	9.1	Introduction, 163	
	9.2	Research Context, 164	•
	9.3	Univariate ANCOVA, 166	
		<ul><li>9.3.1 Testing for Equality of Regression Slopes, 166</li><li>9.3.2 Omnibus Test of Adjusted Means, 168</li></ul>	
	9.4	Multivariate ANCOVA (MANCOVA), 170	
	,,,	9.4.1 Matrix Calculations, 170	
		9.4.2 Testing for Equal Slopes, 171	
	9.5	Computer Application I, 173	
	9.6	Comparing Adjusted Means—Omnibus Test, 174	
	9.7	Computer Application II, 175	
	9.8	Contrast Analysis, 180	
	9.9	Computer Application III, 180	

CONTENTS	xii

CON	(ILIVID		AIII
	9.10	Summary, 184 Technical Note, 184	
		Exercises, 190	
10	Repe	eated-Measures Analysis	193
	10.1	Introduction, 193	
	10.2	Research Context, 195	
	10.3	Univariate Analyses, 196	
		10.3.1 Omnibus Test, 196 10.3.2 Contrast Analysis, 197	
	10.4	Multivariate Analysis, 199	
		Computer Application I, 202	
		Univariate and Multivariate Analyses, 204	
		Testing for Sphericity, 207	
		Computer Application II, 210	
		Contrast Analysis, 212	
		Computer Application III, 214	
	10.11	Summary, 216	
		Technical Notes, 217	
		Exercises, 223	
11	Misso	d Madal Analysis	225
11		d-Model Analysis	227
	11.1	Introduction, 227	
		Research Context, 228	
	11.3	Univariate Analysis, 229	
	11.4	Multivariate Analysis, 231	
		11.4.1 Group-by-Time Interaction, 232 11.4.2 Repeated-Measures Variable Main Effect, 235	
	11.5	Computer Application I, 237	
		Contrast Analysis, 240	
	11.7	Computer Application II, 243	
	11.8	Summary, 246	
		Technical Note, 247	
		Exercises, 249	
IV	GRO	UP MEMBERSHIP PREDICTION	253
12	Class	ification Basics	255
	12.1	Introduction, 255	
	12.2	Notion of Distance, 256	

xiv CONTENTS

	12.3	Distance and Classification, 259	
	12.4	Classification Rules in General, 260	
		12.4.1 Maximum Likelihood, 260	
		12.4.2 Typicality Probability, 261	
		12.4.3 Posterior Probability, 262	
		12.4.4 Prior Probability, 263	
	12.5	Comments, 264	
		Technical Note, 265	
		Further Reading, 265	
		Exercises, 266	
13	Mult	ivariate Normal Rules	269
	13.1	Introduction, 269	
	13.2	Normal Density Functions, 269	
		Classification Rules Based on Normality, 271	
		Classification Functions, 273	
		13.4.1 Quadratic Functions, 273	
		13.4.2 Linear Functions, 274	
		13.4.3 Distance-Based Classification, 275	
	13.5	Summary of Classification Statistics, 277	
	13.6	Choice of Rule Form, 278	
		13.6.1 Normal-Based Rule, 278	
		13.6.2 Covariance Matrix Equality, 279	
		13.6.3 Rule Choice, 280	
		13.6.4 Priors, 281	
	13.7	Comments, 281	
		Technical Notes, 283	
		Further Reading, 283	
		Exercises, 284	
14	Class	sification Results	285
	14.1	Introduction, 285	
	14.2	Research Context, 285	
	14.3	Computer Application, 286	
	14.4	Individual Unit Results, 287	
		14.4.1 In-Doubt Units, 288	
		14.4.2 Outliers, 289	
	14.5	Group Results, 290	

CONTENTS

14.6 Comments, 291

		Technical Note, 291	
		Exercises, 292	
15	Hit F	Rate Estimation	295
	15.1	Introduction, 295	
	15.2	True Hit Rates, 296	
	15.3	Hit Rate Estimators, 297	
		<ul><li>15.3.1 Formula Estimators, 297</li><li>15.3.2 Internal Analysis, 299</li></ul>	
		15.3.3 External Analysis, 300	
		15.3.4 Maximum-Posterior-Probability Method, 302	
		Computer Application, 304	
	15.5	Choice of Hit Rate Estimator, 306	
	15.6	Outliers and In-Doubt Units, 306	
		15.6.1 Outliers, 307	
		15.6.2 In-Doubt Units, 307	
	15.7	Sample Size, 309	
	15.8	Comments, 310	
		Further Reading, 311	
		Exercises, 312	
		1	
16	Effec	ctiveness of Classification Rules	315
	16.1	Introduction, 315	
	16.2	Proportional Chance Criterion, 316	
		16.2.1 Definition, 316	
		16.2.2 Statistical Test, 317	
	16.3	Maximum-Chance Criterion, 319	
	16.4	Improvement over Chance, 320	
	16.5	Comparison of Rules, 320	
	16.6	Computer Application I, 321	
	16.7	Effect of Unequal Priors, 323	
	16.8	PDA Validity/Reliability, 325	
	16.9	Applying a Classification Rule to New Units, 325	
		16.9.1 Computer Application II, 326	
		16.9.2 Computer Application III, 327	
	16.10	Comments, 330	
		Technical Notes, 330	
		Further Reading, 331	
		Exercises, 332	

xvi CONTENTS

17	Delet	ing and Ordering Predictors	335
	17.1	Introduction, 335	
	17.2	Predictor Deletion, 336	
		<ul> <li>17.2.1 Purposes of Deletion, 336</li> <li>17.2.2 Deletion Methods, 336</li> <li>17.2.3 Package Analyses, 337</li> <li>17.2.4 All Possible Subsets, 337</li> </ul>	
	17.3	Computer Application, 337	
	17.4	Predictor Ordering, 340	
		<ul><li>17.4.1 Meaning of Importance, 340</li><li>17.4.2 Variable Ranking, 340</li></ul>	
	17.5	Reanalysis, 343	
	17.6	Comments, 343	
	17.7	Side Note, 345	
		Further Reading, 346	
		Exercises, 347	
18	Two-	Group Classification	349
	18.1	Introduction, 349	
	18.2	Two-Group Rule, 349	
	18.3	Regression Analogy, 351	
	18.4	MRA-PDA Relationship, 353	
	18.5	Necessary Sample Size, 355	
	18.6	Univariate Classification, 356	
		Further Reading, 357	
		Exercises, 359	
19	Noni	normal Rules	361
	19.1	Introduction, 361	
	19.2	Continuous Variables, 362	
		<ul> <li>19.2.1 Rank Transformation Analysis, 362</li> <li>19.2.2 Nearest-Neighbor Analyses, 363</li> <li>19.2.3 Another Density Estimation Analysis, 366</li> <li>19.2.4 Other Analyses, 366</li> </ul>	
	19.3	Categorical Variables, 366	
		<ul> <li>19.3.1 Direct Probability Estimation Analysis, 367</li> <li>19.3.2 Dummy Variable Analysis, 367</li> <li>19.3.3 Overall–Woodward Analysis, 368</li> <li>19.3.4 Fisher–Lancaster Analysis, 368</li> <li>19.3.5 Other Analysis, 368</li> </ul>	
	10.4	19.3.5 Other Analyses, 369	
	19.4	Predictor Mixtures, 369	

CONTENTS		xvii
----------	--	------

	19.5	Comments, 370	
		Further Reading, 371	
		Exercises, 373	
20	Repo	rting PDA Results	375
	20.1	Introduction, 375	
	20.2	Example of Reporting PDA Results, 375	
		Some Additional Specific PDA Information, 378	
	20.4	Computer Package Information, 379	
	20.5	Reporting Terms, 379	
	20.6	Sources of PDA Applications, 381	
	20.7	Concerns, 381	
	20.8	Overview, 382	
		Further Reading, 383	
		Exercises, 383	
21	PDA	-Related Analyses	385
	21.1	Introduction, 385	
	21.2	Nonlinear Methods, 385	
		21.2.1 Classification and Regression Trees (CART), 385	
		21.2.2 Logistic Regression, 385	
		21.2.3 Neural Networks, 386	
	21.3	Other Methods, 386	
		21.3.1 Cluster Analysis, 386	
		21.3.2 Image Analysis, 387 21.3.3 Optimal Allocation, 387	
		21.3.4 Pattern Recognition, 387	
		Further Reading, 388	
$\mathbf{V}$	ISSU	ES AND PROBLEMS	391
22		t nn a land	202
22		s in PDA and DDA	393
	22.1	Introduction, 393	
	22.2	Five Choices in PDA, 393	
		<ul><li>22.2.1 Linear Versus Quadratic Rules, 393</li><li>22.2.2 Nonnormal Classification Rules, 394</li></ul>	
		22.2.2 Nonhormal Classification Rules, 394 22.2.3 Prior Probabilities, 394	
		22.2.4 Misclassification Costs, 394	
		22.2.5 Hit-Rate Estimation, 395	
	22.3	Stepwise Analyses, 395	
	22.4	Standardized Weights Versus Structure r's, 396	

xviii CONTENTS

22.5 Data-Based Structure, 398 Further Reading, 400

23	Prob	lems in PDA and DDA	401
	23.1	Introduction, 401	
	23.2	Missing Data, 401	
		23.2.1 Data Inspection, 401	
		23.2.2 Data Imputation, 402	
		23.2.3 Missing <i>G</i> Values, 404	
		23.2.4 Ad Hoc Strategy, 404	
	23.3	Outliers and Influential Observations, 405	
		23.3.1 Outlier Identification, 405	
		23.3.2 Influential Observations, 406	
	23.4	Initial Group Misclassification, 406	
	23.5	Misclassification Costs, 407	
	23.6	Statistical Versus Clinical Prediction, 407	
	23.7	Other Problems, 409	
		Further Reading, 409	
Ap	pendix	A Data Set Descriptions	411
Appendix B Some DA-Related Originators		B Some DA-Related Originators	415
Ap	pendix	C List of Computer Syntax	419
Appendix D Contents of Wiley Website		A D Contents of Wiley Website	421
References			425
Answers to Exercises			449
Index			481