

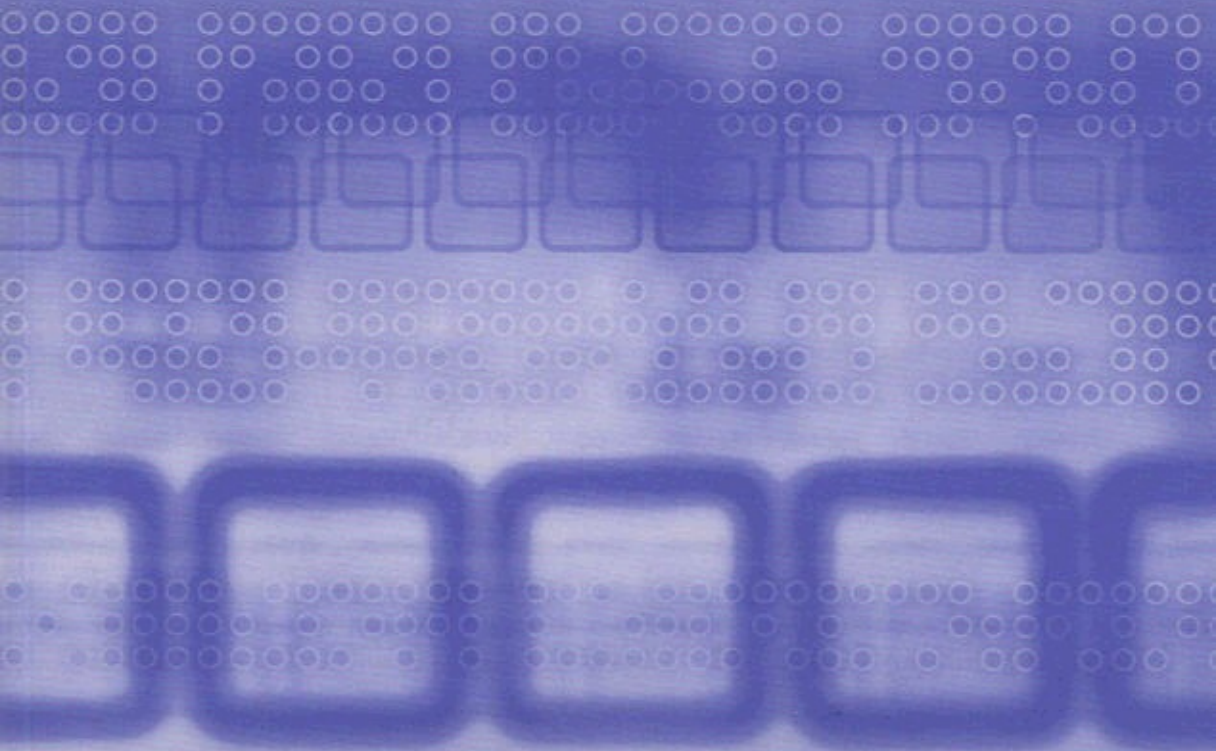
OXFORD
LOGIC

OXFORD TEXTS IN LOGIC 1

A First Course in Logic

*An Introduction to Model Theory,
Proof Theory, Computability,
and Complexity*

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