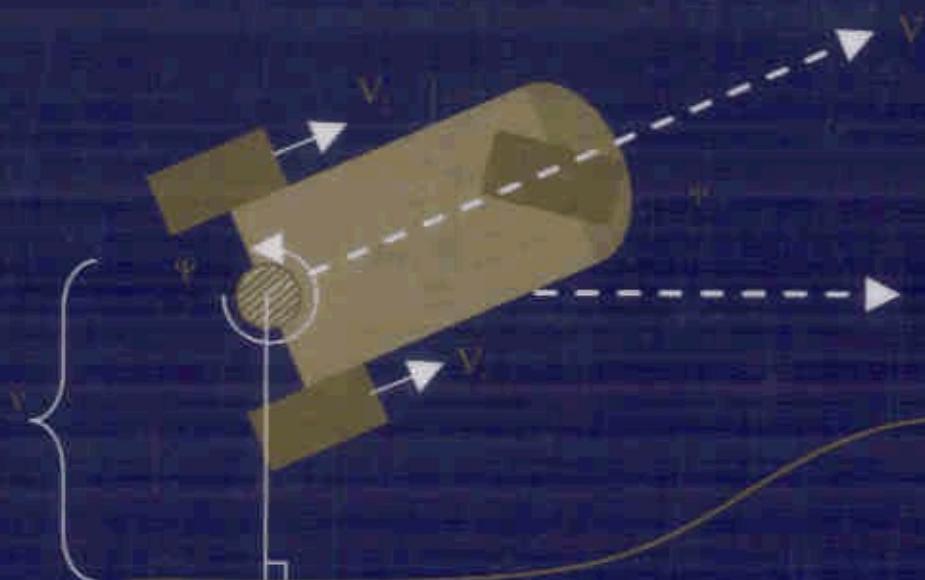


# Adaptive Control Tutorial



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Advances in Design and Control

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