



Pearson International Edition

OPERATIONS RESEARCH

AN INTRODUCTION
EIGHTH EDITION

HAMDY A. TAHA

Featuring AMPL, Solver,
Excel and TORA

Contents

Preface xv

About the Author xvii

Trademarks xviii

Chapter 1 What Is Operations Research? 1

- 1.1 Operations Research Models 1
- 1.2 Solving the OR Model 4
- 1.3 Queuing and Simulation Models 5
- 1.4 More Than Just Mathematics 5
- 1.5 About This Book 7
- References 7

Chapter 2 Modeling with Linear Programming 9

- 2.1 Two-Variable LP Model 10
- 2.2 Graphical LP Solution 13
 - 2.2.1 Solution of a Maximization Model 14
 - 2.2.2 Solution of a Minimization Model 21
- 2.3 Selected LP Applications 25
 - 2.3.1 Urban Planning 25
 - 2.3.2 Currency Arbitrage 30
 - 2.3.3 Investment 35
 - 2.3.4 Production Planning and Inventory Control 40
 - 2.3.5 Blending and Refining 49
 - 2.3.6 Manpower Planning 55
 - 2.3.7 Additional Applications 58
- 2.4 Computer Solution with Solver and AMPL 66
 - 2.4.1 LP Solution with Excel Solver 66
 - 2.4.2 LP Solution with AMPL 70
 - References 78

Chapter 3 The Simplex Method and Sensitivity Analysis 79

- 3.1 LP Model in Equation Form 80
 - 3.1.1 Converting Inequalities into Equations with Nonnegative Right-Hand Side 80
 - 3.1.2 Dealing with Unrestricted Variables 82
- 3.2 Transition from Graphical to Algebraic Solution 83

- 3.3 The Simplex Method 88**
 - 3.3.1 Iterative Nature of the Simplex Method 88
 - 3.3.2 Computational Details of the Simplex Algorithm 91
 - 3.3.3 Summary of the Simplex Method 97
- 3.4 Artificial Starting Solution 101**
 - 3.4.1 *M*-Method 101
 - 3.4.2 Two-Phase Method 106
- 3.5 Special Cases in the Simplex Method 110**
 - 3.5.1 Degeneracy 110
 - 3.5.2 Alternative Optima 114
 - 3.5.3 Unbounded Solution 116
 - 3.5.4 Infeasible Solution 118
- 3.6 Sensitivity Analysis 120**
 - 3.6.1 Graphical Sensitivity Analysis 121
 - 3.6.2 Algebraic Sensitivity Analysis—Changes in the Right-Hand Side 126
 - 3.6.3 Algebraic Sensitivity Analysis—Objective Function 137
 - 3.6.4 Sensitivity Analysis with TORA, Solver, and AMPL 144
- References 147**

Chapter 4 Duality and Post-Optimal Analysis 149

- 4.1 Definition of the Dual Problem 149**
- 4.2 Primal-Dual Relationships 154**
 - 4.2.1 Review of Simple Matrix Operations 154
 - 4.2.2 Simplex Tableau Layout 156
 - 4.2.3 Optimal Dual Solution 157
 - 4.2.4 Simplex Tableau Computations 163
- 4.3 Economic Interpretation of Duality 167**
 - 4.3.1 Economic Interpretation of Dual Variables 168
 - 4.3.2 Economic Interpretation of Dual Constraints 170
- 4.4 Additional Simplex Algorithms 172**
 - 4.4.1 Dual Simplex Algorithm 172
 - 4.4.2 Generalized Simplex Algorithm 178
- 4.5 Post-Optimal Analysis 179**
 - 4.5.1 Changes Affecting Feasibility 180
 - 4.5.2 Changes Affecting Optimality 185
- References 189**

Chapter 5 Transportation Model and Its Variants 191

- 5.1 Definition of the Transportation Model 192**
- 5.2 Nontraditional Transportation Models 199**
- 5.3 The Transportation Algorithm 204**
 - 5.3.1 Determination of the Starting Solution 205
 - 5.3.2 Iterative Computations of the Transportation Algorithm 209

- 5.3.3 Simplex Method Explanation of the Method of Multipliers 217
- 5.4 The Assignment Model 219
 - 5.4.1 The Hungarian Method 220
 - 5.4.2 Simplex Explanation of the Hungarian Method 225
- 5.5 The Transshipment Model 226
 - References 230

Chapter 6 Network Models 231

- 6.1 Scope and Definition of Network Models 232
- 6.2 Minimal Spanning Tree Algorithm 235
- 6.3 Shortest-Route Problem 239
 - 6.3.1 Examples of the Shortest-Route Applications 239
 - 6.3.2 Shortest-Route Algorithms 243
 - 6.3.3 Linear Programming Formulation of the Shortest-Route Problem 252
- 6.4 Maximal flow model 258
 - 6.4.1 Enumeration of Cuts 259
 - 6.4.2 Maximal Flow Algorithm 260
 - 6.4.3 Linear Programming Formulation of Maximal Flow Mode 268
- 6.5 CPM and PERT 272
 - 6.5.1 Network Representation 272
 - 6.5.2 Critical Path (CPM) Computations 277
 - 6.5.3 Construction of the Time Schedule 281
 - 6.5.4 Linear Programming Formulation of CPM 287
 - 6.5.5 PERT Networks 288
 - References 291

Chapter 7 Advanced Linear Programming 293

- 7.1 Simplex Method Fundamentals 294
 - 7.1.1 From Extreme Points to Basic Solutions 296
 - 7.1.2 Generalized Simplex Tableau in Matrix Form 299
- 7.2 Revised Simplex Method 302
 - 7.2.1 Development of the Optimality and Feasibility Conditions 303
 - 7.2.2 Revised Simplex Algorithm 305
- 7.3 Bounded-Variables Algorithm 311
- 7.4 Duality 317
 - 7.4.1 Matrix Definition of the Dual Problem 318
 - 7.4.2 Optimal Dual Solution 318
- 7.5 Parametric Linear Programming 322
 - 7.5.1 Parametric Changes in C 323
 - 7.5.2 Parametric Changes in b 326
 - References 328

Chapter 8 Goal Programming 329

- 8.1 A Goal Programming Formulation 330
- 8.2 Goal Programming Algorithms 334
 - 8.2.1 The Weights Method 334
 - 8.2.2 The Preemptive Method 337
- References 344

Chapter 9 Integer Linear Programming 345

- 9.1 Illustrative Applications 346
 - 9.1.1 Capital Budgeting 346
 - 9.1.2 Set-Covering Problem 350
 - 9.1.3 Fixed-Charge Problem 356
 - 9.1.4 Either-Or and If-Then Constraints 360
- 9.2 Integer Programming Algorithms 365
 - 9.2.1 Branch-and-Bound (B&B) Algorithm 366
 - 9.2.2 Cutting-Plane Algorithm 375
 - 9.2.3 Computational Considerations in ILP 380
- 9.3 Traveling Salesperson (TSP) Problem 381
 - 9.3.1 Heuristic Algorithms 385
 - 9.3.2 B&B Solution Algorithm 388
 - 9.3.3 Cutting-Plane Algorithm 391
- References 393

Chapter 10 Deterministic Dynamic Programming 395

- 10.1 Recursive Nature of Computations in DP 396
- 10.2 Forward and Backward Recursion 399
- 10.3 Selected DP Applications 401
 - 10.3.1 Knapsack/Fly-Away/Cargo-Loading Model 401
 - 10.3.2 Work-Force Size Model 409
 - 10.3.3 Equipment Replacement Model 412
 - 10.3.4 Investment Model 416
 - 10.3.5 Inventory Models 419
- 10.4 Problem of Dimensionality 420
- References 422

Chapter 11 Deterministic Inventory Models 423

- 11.1 General Inventory Model 423
- 11.2 Role of Demand in the Development of Inventory Models 424
- 11.3 Static Economic-Order-Quantity (EOQ) Models 426
 - 11.3.1 Classic EOQ model 426
 - 11.3.2 EOQ with Price Breaks 432
 - 11.3.3 Multi-Item EOQ with Storage Limitation 436
- 11.4 Dynamic EOQ Models 439
 - 11.4.1 No-Setup Model 441
 - 11.4.2 Setup Model 445
- References 457

Chapter 12 Review of Basic Probability 459

- 12.1 Laws of Probability 459
 - 12.1.1 Addition Law of Probability 460
 - 12.1.2 Conditional Law of Probability 461
- 12.2 Random Variables and Probability Distributions 463
- 12.3 Expectation of a Random Variable 465
 - 12.3.1 Mean and Variance (Standard Deviation) of a Random Variable 466
 - 12.3.2 Mean and Variance of Joint Random Variables 468
- 12.4 Four Common Probability Distributions 471
 - 12.4.1 Binomial Distribution 471
 - 12.4.2 Poisson Distribution 472
 - 12.4.3 Negative Exponential Distribution 473
 - 12.4.4 Normal Distribution 474
- 12.5 Empirical Distributions 477
 - References 484

Chapter 13 Decision Analysis and Games 485

- 13.1 Decision Making under Certainty—Analytic Hierarchy Process (AHP) 486
- 13.2 Decision Making under Risk 496
 - 13.2.1 Decision Tree-Based Expected Value Criterion 496
 - 13.2.2 Variations of the Expected Value Criterion 502
- 13.3 Decision under Uncertainty 511
- 13.4 Game Theory 516
 - 13.4.1 Optimal Solution of Two-Person Zero-Sum Games 517
 - 13.4.2 Solution of Mixed Strategy Games 520
 - References 526

Chapter 14 Probabilistic Inventory Models 527

- 14.1 Continuous Review Models 528
 - 14.1.1 “Probabilitized” EOQ Model 528
 - 14.1.2 Probabilistic EOQ Model 530
- 14.2 Single-Period Models 535
 - 14.2.1 No-Setup Model (Newsvendor Model) 535
 - 14.2.2 Setup Model (s - S Policy) 539
- 14.3 Multiperiod Model 541
 - References 544

Chapter 15 Queuing Systems 545

- 15.1 Why Study Queues? 546
- 15.2 Elements of a Queuing Model 547
- 15.3 Role of Exponential Distribution 549
- 15.4 Pure Birth and Death Models (Relationship Between the Exponential and Poisson Distributions) 552
 - 15.4.1 Pure Birth Model 552
 - 15.4.2 Pure Death Model 556

- 15.5 Generalized Poisson Queuing Model 559
- 15.6 Specialized Poisson Queues 564
 - 15.6.1 Steady-State Measures of Performance 565
 - 15.6.2 Single-Server Models 569
 - 15.6.3 Multiple-Server Models 578
 - 15.6.4 Machine Servicing Model—(M/M/R): (GD/K/K), $R < K$ 588
- 15.7 (M/G/1):(GD/∞/∞)—Pollaczek-Khintchine (P-K) Formula 591
- 15.8 Other Queuing Models 593
- 15.9 Queuing Decision Models 593
 - 15.9.1 Cost Models 594
 - 15.9.2 Aspiration Level Model 598
- References 600

Chapter 16 Simulation Modeling 601

- 16.1 Monte Carlo Simulation 601
- 16.2 Types of Simulation 606
- 16.3 Elements of Discrete-Event Simulation 607
 - 16.3.1 Generic Definition of Events 607
 - 16.3.2 Sampling from Probability Distributions 609
- 16.4 Generation of Random Numbers 618
- 16.5 Mechanics of Discrete Simulation 620
 - 16.5.1 Manual Simulation of a Single-Server Model 620
 - 16.5.2 Spreadsheet-Based Simulation of the Single-Server Model 626
- 16.6 Methods for Gathering Statistical Observations 629
 - 16.6.1 Subinterval Method 630
 - 16.6.2 Replication Method 631
 - 16.6.3 Regenerative (Cycle) Method 632
- 16.7 Simulation Languages 634
- References 636

Chapter 17 Markov Chains 637

- 17.1 Definition of a Markov Chain 637
- 17.2 Absolute and n -Step Transition Probabilities 640
- 17.3 Classification of the States in a Markov Chain 642
- 17.4 Steady-State Probabilities and Mean Return Times of Ergodic Chains 645
- 17.5 First Passage Time 650
- 17.6 Analysis of Absorbing States 654
- References 659

Chapter 18 Classical Optimization Theory 661

- 18.1 Unconstrained Problems 661
 - 18.1.1 Necessary and Sufficient Conditions 662
 - 18.1.2 The Newton-Raphson Method 666

- 18.2 Constrained Problems 668**
 - 18.2.1 Equality Constraints 669
 - 18.2.2 Inequality Constraints—Karush-Kuhn-Tucker (KKT) Conditions 681
- References 686

Chapter 19 Nonlinear Programming Algorithms 687

- 19.1 Unconstrained Algorithms 687**
 - 19.1.1 Direct Search Method 687
 - 19.1.2 Gradient Method 691
- 19.2 Constrained Algorithms 695**
 - 19.2.1 Separable Programming 695
 - 19.2.2 Quadratic Programming 704
 - 19.2.3 Chance-Constrained Programming 709
 - 19.2.4 Linear Combinations Method 714
 - 19.2.5 SUMT Algorithm 717
- References 718

Appendix A AMPL Modeling Language 719

- A.1 Rudimentary AMPL Model 719**
- A.2 Components of AMPL Model 720**
- A.3 Mathematical Expressions and Computed Parameters 728**
- A.4 Subsets and Indexed Sets 731**
- A.5 Accessing External Files 733**
 - A.5.1 Simple Read Files 733
 - A.5.2 Using Print and Printf to Retrieve Output 735
 - A.5.3 Input Table Files 735
 - A.5.4 Output Table Files 738
 - A.5.5 Spreadsheet Input/Output Tables 740
- A.6 Interactive Commands 740**
- A.7 Iterative and Conditional Execution of AMPL Commands 742**
- A.8 Sensitivity Analysis Using AMPL 744**
- References 744

Appendix B Statistical Tables 745

Appendix C Partial Solutions to Selected Problems 749

Index 799