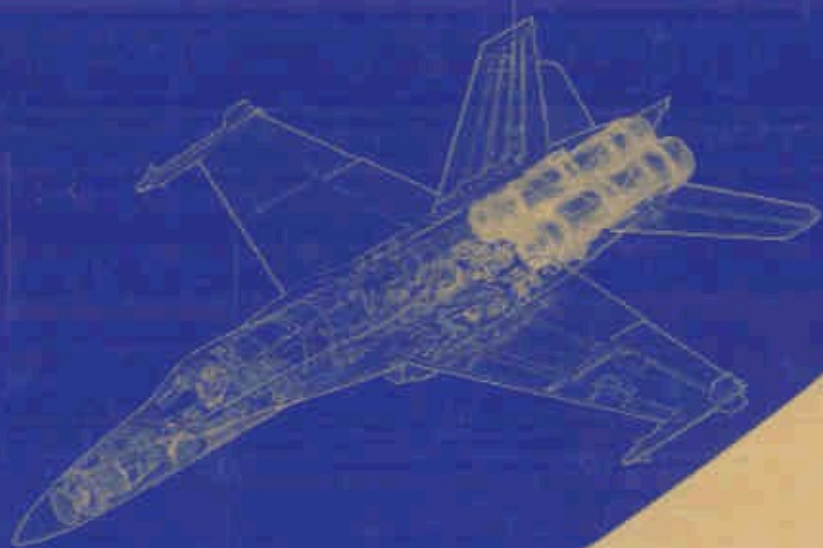


Analysis of Aircraft Structures

An Introduction

SECOND EDITION

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CAMBRIDGE AEROSPACE SERIES

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