

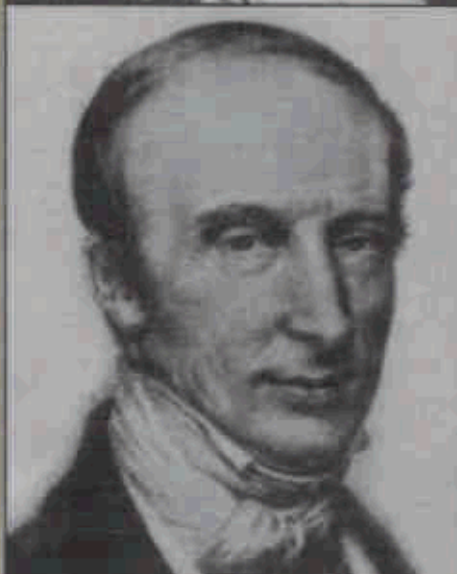
A Radical Approach to Real Analysis

| *Second Edition* |

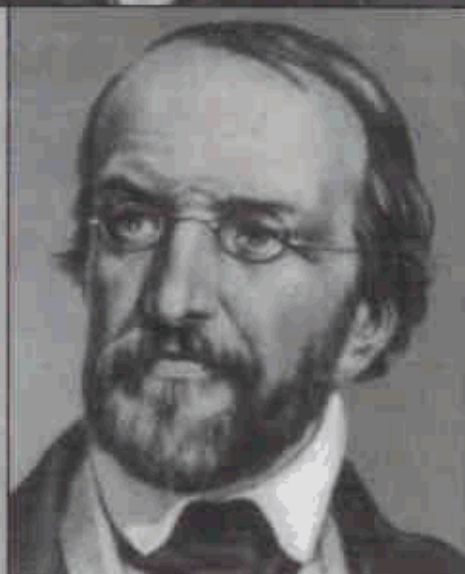
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