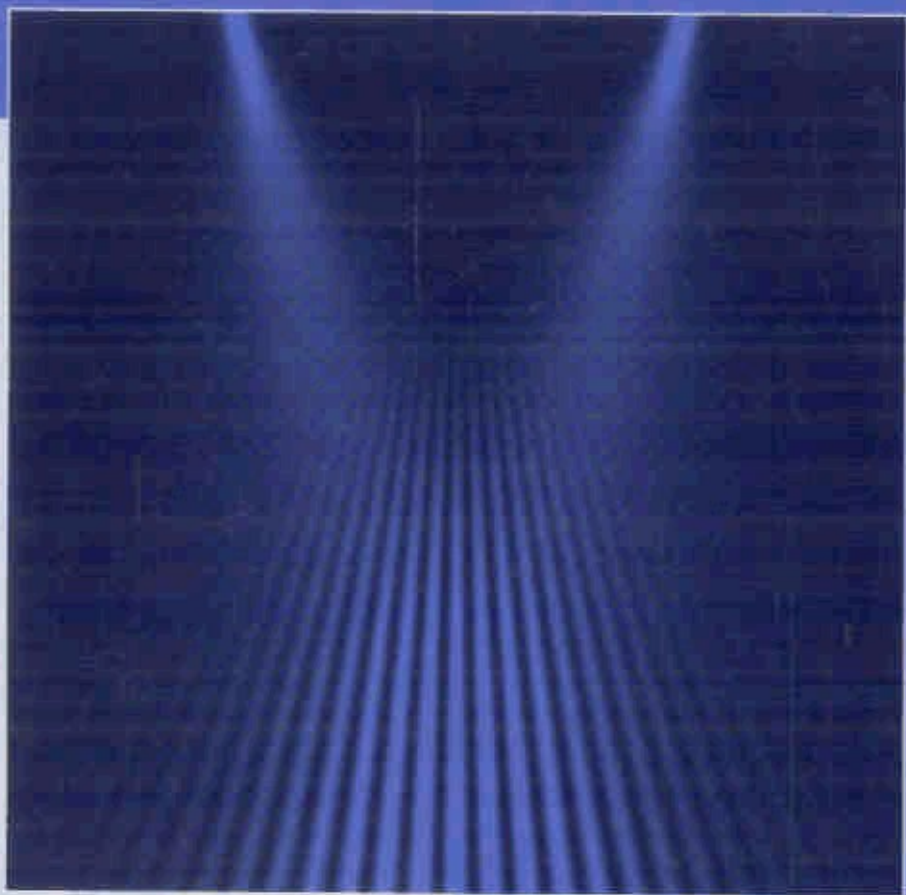


REVISED EDITION

A FIRST COURSE IN

FOURIER ANALYSIS



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CAMBRIDGE

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