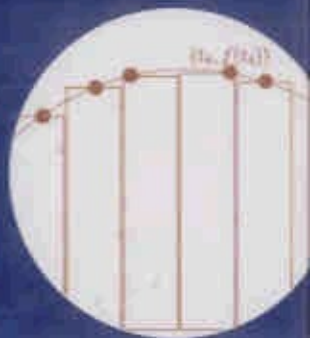
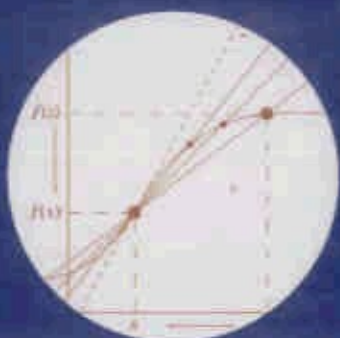
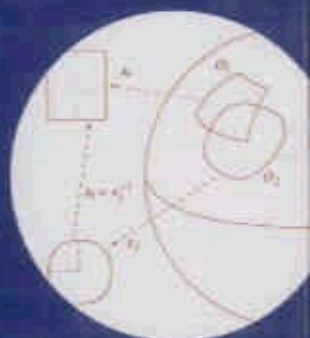
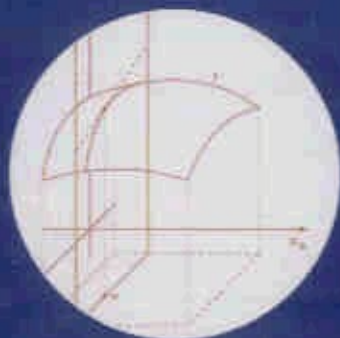
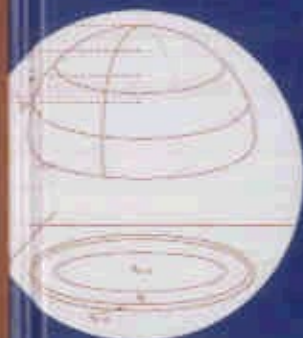




MATHEMATICAL ANALYSIS

A Concise Introduction



Contents

Table of Contents	v
Preface	xi
Part I: Analysis of Functions of a Single Real Variable	
1 The Real Numbers	1
1.1 Field Axioms	1
1.2 Order Axioms	4
1.3 Lowest Upper and Greatest Lower Bounds	8
1.4 Natural Numbers, Integers, and Rational Numbers	11
1.5 Recursion, Induction, Summations, and Products	17
2 Sequences of Real Numbers	25
2.1 Limits	25
2.2 Limit Laws	30
2.3 Cauchy Sequences	36
2.4 Bounded Sequences	40
2.5 Infinite Limits	44
3 Continuous Functions	49
3.1 Limits of Functions	49
3.2 Limit Laws	52
3.3 One-Sided Limits and Infinite Limits	56
3.4 Continuity	59
3.5 Properties of Continuous Functions	66
3.6 Limits at Infinity	69
4 Differentiable Functions	71
4.1 Differentiability	71
4.2 Differentiation Rules	74
4.3 Rolle's Theorem and the Mean Value Theorem	80

5	The Riemann Integral I	85
5.1	Riemann Sums and the Integral	85
5.2	Uniform Continuity and Integrability of Continuous Functions	91
5.3	The Fundamental Theorem of Calculus	95
5.4	The Darboux Integral	97
6	Series of Real Numbers I	101
6.1	Series as a Vehicle To Define Infinite Sums	101
6.2	Absolute Convergence and Unconditional Convergence	108
7	Some Set Theory	117
7.1	The Algebra of Sets	117
7.2	Countable Sets	122
7.3	Uncountable Sets	124
8	The Riemann Integral II	127
8.1	Outer Lebesgue Measure	127
8.2	Lebesgue's Criterion for Riemann Integrability	131
8.3	More Integral Theorems	136
8.4	Improper Riemann Integrals	140
9	The Lebesgue Integral	145
9.1	Lebesgue Measurable Sets	147
9.2	Lebesgue Measurable Functions	153
9.3	Lebesgue Integration	158
9.4	Lebesgue Integrals versus Riemann Integrals	165
10	Series of Real Numbers II	169
10.1	Limits Superior and Inferior	169
10.2	The Root Test and the Ratio Test	172
10.3	Power Series	175
11	Sequences of Functions	179
11.1	Notions of Convergence	179
11.2	Uniform Convergence	182
12	Transcendental Functions	189
12.1	The Exponential Function	189
12.2	Sine and Cosine	193
12.3	L'Hôpital's Rule	199
13	Numerical Methods	203
13.1	Approximation with Taylor Polynomials	204
13.2	Newton's Method	208
13.3	Numerical Integration	214

Part II: Analysis in Abstract Spaces

14	Integration on Measure Spaces	225
14.1	Measure Spaces	225
14.2	Outer Measures	230
14.3	Measurable Functions	234
14.4	Integration of Measurable Functions	235
14.5	Monotone and Dominated Convergence	238
14.6	Convergence in Mean, in Measure, and Almost Everywhere	242
14.7	Product σ -Algebras	245
14.8	Product Measures and Fubini's Theorem	251
15	The Abstract Venues for Analysis	255
15.1	Abstraction I: Vector Spaces	255
15.2	Representation of Elements: Bases and Dimension	259
15.3	Identification of Spaces: Isomorphism	262
15.4	Abstraction II: Inner Product Spaces	264
15.5	Nicer Representations: Orthonormal Sets	267
15.6	Abstraction III: Normed Spaces	269
15.7	Abstraction IV: Metric Spaces	275
15.8	L^p Spaces	278
15.9	Another Number Field: Complex Numbers	281
16	The Topology of Metric Spaces	287
16.1	Convergence of Sequences	287
16.2	Completeness	291
16.3	Continuous Functions	296
16.4	Open and Closed Sets	301
16.5	Compactness	309
16.6	The Normed Topology of $\mathbb{R}^d$	316
16.7	Dense Subspaces	322
16.8	Connectedness	330
16.9	Locally Compact Spaces	333
17	Differentiation in Normed Spaces	341
17.1	Continuous Linear Functions	342
17.2	Matrix Representation of Linear Functions	348
17.3	Differentiability	353
17.4	The Mean Value Theorem	360
17.5	How Partial Derivatives Fit In	362
17.6	Multilinear Functions (Tensors)	369
17.7	Higher Derivatives	373
17.8	The Implicit Function Theorem	380

18 Measure, Topology, and Differentiation	385
18.1 Lebesgue Measurable Sets in $\mathbb{R}^d$	385
18.2 C^∞ and Approximation of Integrable Functions	391
18.3 Tensor Algebra and Determinants	397
18.4 Multidimensional Substitution	407
19 Introduction to Differential Geometry	421
19.1 Manifolds	421
19.2 Tangent Spaces and Differentiable Functions	427
19.3 Differential Forms, Integrals Over the Unit Cube	434
19.4 k -Forms and Integrals Over k -Chains	443
19.5 Integration on Manifolds	452
19.6 Stokes' Theorem	458
20 Hilbert Spaces	463
20.1 Orthonormal Bases	463
20.2 Fourier Series	467
20.3 The Riesz Representation Theorem	475
Part III: Applied Analysis	
21 Physics Background	483
21.1 Harmonic Oscillators	484
21.2 Heat and Diffusion	486
21.3 Separation of Variables, Fourier Series, and Ordinary Differential Equations	490
21.4 Maxwell's Equations	493
21.5 The Navier Stokes Equation for the Conservation of Mass	496
22 Ordinary Differential Equations	505
22.1 Banach Space Valued Differential Equations	505
22.2 An Existence and Uniqueness Theorem	508
22.3 Linear Differential Equations	510
23 The Finite Element Method	513
23.1 Ritz-Galerkin Approximation	513
23.2 Weakly Differentiable Functions	518
23.3 Sobolev Spaces	524
23.4 Elliptic Differential Operators	532
23.5 Finite Elements	536
Conclusion and Outlook	544

Appendices

A	Logic	545
A.1	Statements	545
A.2	Negations	546
B	Set Theory	547
B.1	The Zermelo-Fraenkel Axioms	547
B.2	Relations and Functions	548
C	Natural Numbers, Integers, and Rational Numbers	549
C.1	The Natural Numbers	549
C.2	The Integers	550
C.3	The Rational Numbers	550
	Bibliography	551
	Index	553