

Nonlinear Filtering and Smoothing

An Introduction to Martingales,
Stochastic Integrals and Estimation

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