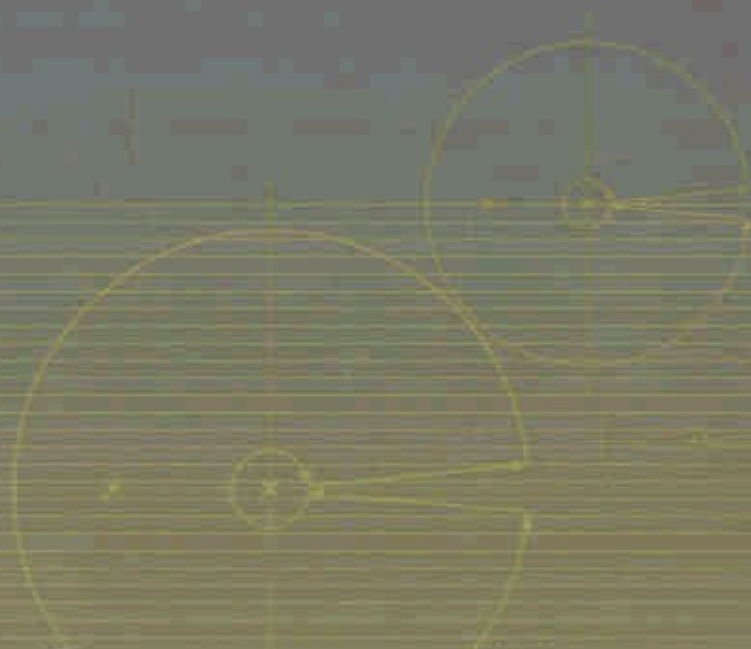


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COMPLEX VARIABLES

WITH APPLICATIONS

Birkhäuser

Contents

Preface	vii
1 Algebraic and Geometric Preliminaries	1
1.1 The Complex Field	1
1.2 Rectangular Representation	5
1.3 Polar Representation	15
2 Topological and Analytic Preliminaries	25
2.1 Point Sets in the Plane	25
2.2 Sequences	32
2.3 Compactness	39
2.4 Stereographic Projection	44
2.5 Continuity	48
3 Bilinear Transformations and Mappings	61
3.1 Basic Mappings	61
3.2 Linear Fractional Transformations	66
3.3 Other Mappings	85
4 Elementary Functions	91
4.1 The Exponential Function	91
4.2 Mapping Properties	100
4.3 The Logarithmic Function	108
4.4 Complex Exponents	114
5 Analytic Functions	121
5.1 Cauchy–Riemann Equation	121
5.2 Analyticity	130
5.3 Harmonic Functions	141

6	Power Series	153
6.1	Sequences Revisited	153
6.2	Uniform Convergence	164
6.3	Maclaurin and Taylor Series	173
6.4	Operations on Power Series	186
7	Complex Integration and Cauchy's Theorem	195
7.1	Curves	195
7.2	Parameterizations	207
7.3	Line Integrals	217
7.4	Cauchy's Theorem	226
8	Applications of Cauchy's Theorem	243
8.1	Cauchy's Integral Formula	243
8.2	Cauchy's Inequality and Applications	263
8.3	Maximum Modulus Theorem	275
9	Laurent Series and the Residue Theorem	285
9.1	Laurent Series	285
9.2	Classification of Singularities	293
9.3	Evaluation of Real Integrals	308
9.4	Argument Principle	331
10	Harmonic Functions	349
10.1	Comparison with Analytic Functions	349
10.2	Poisson Integral Formula	358
10.3	Positive Harmonic Functions	371
11	Conformal Mapping and the Riemann Mapping Theorem ..	379
11.1	Conformal Mappings	379
11.2	Normal Families	390
11.3	Riemann Mapping Theorem	395
11.4	The Class $\mathcal{S}$	405
12	Entire and Meromorphic Functions	411
12.1	Infinite Products	411
12.2	Weierstrass' Product Theorem	422
12.3	Mittag-Leffler Theorem	437
13	Analytic Continuation	445
13.1	Basic Concepts	445
13.2	Special Functions	458
	References and Further Reading	473
	Index of Special Notations	475

Index 479

Hints for Selected Questions and Exercises 485