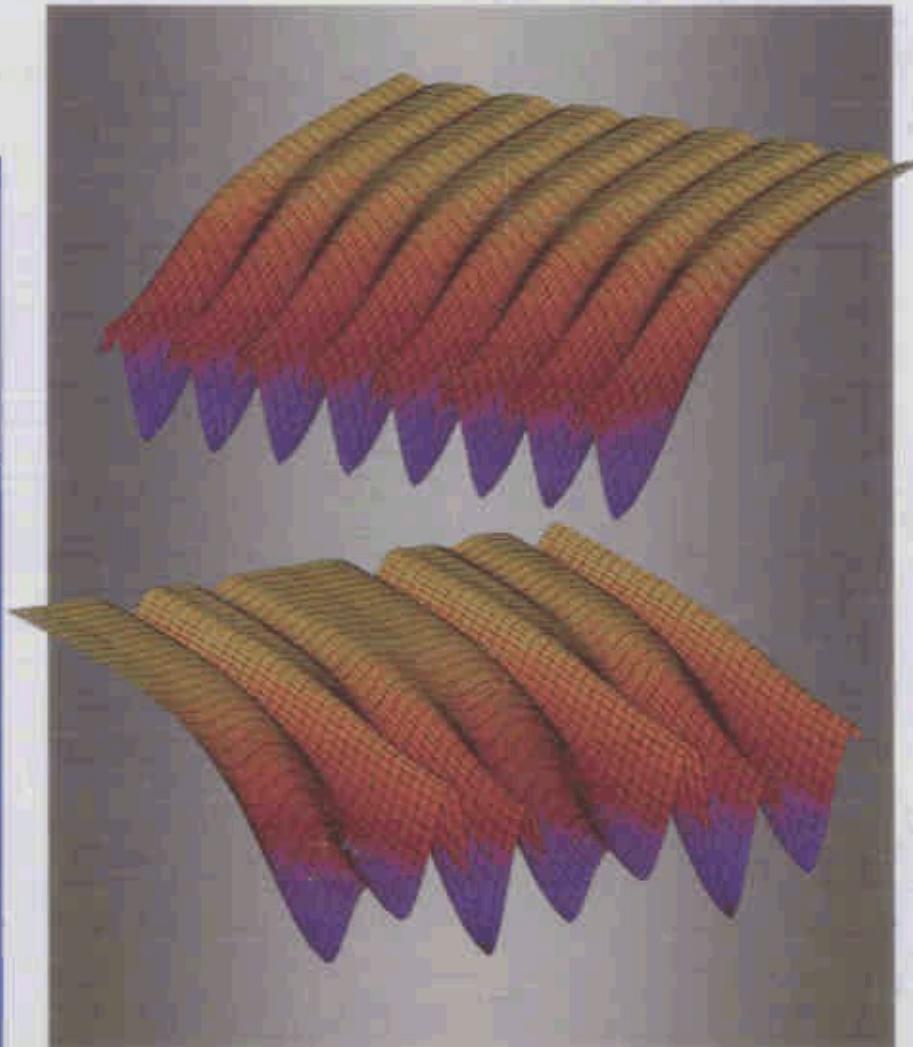


Edited by  
E. Schöll and H. G. Schuster

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# Handbook of Chaos Control

Second, Completely Revised and Enlarged Edition



## Contents

Preface *XXI*

List of Contributors *XXIII*

### Part I Basic Aspects and Extension of Methods

<b>1</b>	<b>Controlling Chaos</b>	3
	<i>Elbert E.N. Macau and Celso Grebogi</i>	
1.1	Introduction	3
1.2	The OGY Chaos Control	6
1.3	Targeting—Steering Chaotic Trajectories	8
1.3.1	Part I: Finding a Proper Trajectory	9
1.3.2	Part II: Finding a Pseudo-Orbit Trajectory	10
1.3.3	The Targeting Algorithm	12
1.4	Applying Control of Chaos and Targeting Ideas	13
1.4.1	Controlling an Electronic Circuit	13
1.4.2	Controlling a Complex System	19
1.5	Conclusion	26
	<i>References</i>	26
<b>2</b>	<b>Time-Delay Control for Discrete Maps</b>	29
	<i>Joshua E.S. Socolar</i>	
2.1	Overview: Why Study Discrete Maps?	29
2.2	Theme and Variations	31
2.2.1	Rudimentary Time-Delay Feedback	32
2.2.2	Extending the Domain of Control	34
2.2.3	High-Dimensional Systems	37
2.3	Robustness of Time-Delay Stabilization	41
2.4	Summary	44
	<i>Acknowledgments</i>	44
	<i>References</i>	44

<b>3</b>	<b>An Analytical Treatment of the Delayed Feedback Control Algorithm</b>	<b>47</b>
	<i>Kestutis Pyragas, Tatjana Pyragiené, and Viktoras Pyragas</i>	
3.1	Introduction	47
3.2	Proportional Versus Delayed Feedback	50
3.3	Controlling Periodic Orbits Arising from a Period Doubling Bifurcation	53
3.3.1	Example: Controlling the Rössler System	54
3.4	Control of Forced Self-Sustained Oscillations	57
3.4.1	Problem Formulation and Averaged Equation	57
3.4.2	Periodic Orbits of the Free System	58
3.4.3	Linear Stability of the System Controlled by Delayed Feedback	60
3.4.4	Numerical Demonstrations	63
3.5	Controlling Torsion-Free Periodic Orbits	63
3.5.1	Example: Controlling the Lorenz System at a Subcritical Hopf Bifurcation	65
3.6	Conclusions	68
	<i>References</i>	70
<b>4</b>	<b>Beyond the Odd-Number Limitation of Time-Delayed Feedback Control</b>	<b>73</b>
	<i>Bernold Fiedler, Valentin Flunkert, Marc Georgi, Philipp Hövel, and Eckehard Schöll</i>	
4.1	Introduction	73
4.2	Mechanism of Stabilization	74
4.3	Conditions on the Feedback Gain	78
4.4	Conclusion	82
	Acknowledgments	82
	Appendix: Calculation of Floquet Exponents	82
	<i>References</i>	83
<b>5</b>	<b>On Global Properties of Time-Delayed Feedback Control</b>	<b>85</b>
	<i>Wolfram Just</i>	
5.1	Introduction	85
5.2	A Comment on Control and Root Finding Algorithms	88
5.3	Codimension-Two Bifurcations and Basins of Attraction	91
5.3.1	The Transition from Super- to Subcritical Behavior	91
5.3.2	Probing Basins of Attraction in Experiments	93
5.4	A Case Study of Global Features for Time-Delayed Feedback Control	94
5.4.1	Analytical Bifurcation Analysis of One-Dimensional Maps	95
5.4.2	Dependence of Sub- and Supercritical Behavior on the Observable	98
5.4.3	Influence of the Coupling of the Control Force	99
5.5	Conclusion	101

Acknowledgments	102
Appendix A. Normal Form Reduction	103
Appendix B. Super- and Subcritical Hopf Bifurcation for Maps	106
References	106
<b>6 Poincaré-Based Control of Delayed Measured Systems:</b>	
<b>Limitations and Improved Control</b>	109
<i>Jens Christian Claussen</i>	
6.1 Introduction	109
6.1.1 The Delay Problem—Time-Discrete Case	109
6.1.2 Experimental Setups with Delay	111
6.2 Ott-Grebogi-Yorke (OGY) Control	112
6.3 Limitations of Unmodified Control and Simple Improved Control Schemes	113
6.3.1 Limitations of Unmodified OGY Control in the Presence of Delay	113
6.3.2 Stability Diagrams Derived by the Jury Criterion	116
6.3.3 Stabilizing Unknown Fixed Points: Limitations of Unmodified Difference Control	116
6.3.4 Rhythmic Control Schemes: Rhythmic OGY Control	119
6.3.5 Rhythmic Difference Control	120
6.3.6 A Simple Memory Control Scheme: Using State Space Memory	122
6.4 Optimal Improved Control Schemes	123
6.4.1 Linear Predictive Logging Control (LPLC)	123
6.4.2 Nonlinear Predictive Logging Control	124
6.4.3 Stabilization of Unknown Fixed Points: Memory Difference Control (MDC)	125
6.5 Summary	126
6.5 References	127
<b>7 Nonlinear and Adaptive Control of Chaos</b>	129
<i>Alexander Fradkov and Alexander Pogromsky</i>	
7.1 Introduction	129
7.2 Chaos and Control: Preliminaries	130
7.2.1 Definitions of Chaos	130
7.2.2 Models of Controlled Systems	131
7.2.3 Control Goals	132
7.3 Methods of Nonlinear Control	134
7.3.1 Gradient Method	135
7.3.2 Speed-Gradient Method	136
7.3.3 Feedback Linearization	141
7.3.4 Other Methods	142
7.3.5 Gradient Control of the Hénon System	144
7.3.6 Feedback Linearization Control of the Lorenz System	146

7.3.7	Speed-Gradient Stabilization of the Equilibrium Point for the Thermal Convection Loop Model	147
7.4	Adaptive Control	148
7.4.1	General Definitions	148
7.4.2	Adaptive Master-Slave Synchronization of Rössler Systems	149
7.5	Other Problems	154
7.6	Conclusions	155
	Acknowledgment	155
	References	156

## Part II Controlling Space-Time Chaos

<b>8</b>	<b>Localized Control of Spatiotemporal Chaos</b>	161
	<i>Roman O. Grigoriev and Andreas Handel</i>	
8.1	Introduction	161
8.1.1	Empirical Control	163
8.1.2	Model-Based Control	164
8.2	Symmetry and the Minimal Number of Sensors/Actuators	167
8.3	Nonnormality and Noise Amplification	170
8.4	Nonlinearity and the Critical Noise Level	175
8.5	Conclusions	177
	References	177
<b>9</b>	<b>Controlling Spatiotemporal Chaos: The Paradigm of the Complex Ginzburg-Landau Equation</b>	181
	<i>Stefano Boccaletti and Jean Bragard</i>	
9.1	Introduction	181
9.2	The Complex Ginzburg-Landau Equation	183
9.2.1	Dynamics Characterization	185
9.3	Control of the CGLE	187
9.4	Conclusions and Perspectives	192
	Acknowledgment	193
	References	193
<b>10</b>	<b>Multiple Delay Feedback Control</b>	197
	<i>Alexander Ahlborn and Ulrich Parlitz</i>	
10.1	Introduction	197
10.2	Multiple Delay Feedback Control	198
10.2.1	Linear Stability Analysis	199
10.2.2	Example: Colpitts Oscillator	200
10.2.3	Comparison with High-Pass Filter and PD Controller	203
10.2.4	Transfer Function of MDFC	204
10.3	From Multiple Delay Feedback Control to Notch Filter Feedback	206
10.4	Controllability Criteria	208

10.4.1	Multiple Delay Feedback Control	209
10.4.2	Notch Filter Feedback and High-Pass Filter	210
10.5	Laser Stabilization Using MDFC and NFF	211
10.6	Controlling Spatiotemporal Chaos	213
10.6.1	The Ginzburg-Landau Equation	213
10.6.2	Controlling Traveling Plane Waves	214
10.6.3	Local Feedback Control	215
10.7	Conclusion	218
	References	219

### **Part III** Controlling Noisy Motion

<b>11</b>	<b>Control of Noise-Induced Dynamics</b>	223
	<i>Natalia B. Janson, Alexander G. Balanov, and Eckehard Schöll</i>	
11.1	Introduction	223
11.2	Noise-Induced Oscillations Below Andronov-Hopf Bifurcation and their Control	226
11.2.1	Weak Noise and Control: Correlation Function	228
11.2.2	Weak Noise and No Control: Correlation Time and Spectrum	229
11.2.3	Weak Noise and Control: Correlation Time	231
11.2.4	Weak Noise and Control: Spectrum	235
11.2.5	Any Noise and No Control: Correlation Time	236
11.2.6	Any Noise and Control: Correlation Time and Spectrum	238
11.2.7	So, What Can We Control?	240
11.3	Noise-Induced Oscillations in an Excitable System and their Control	241
11.3.1	Coherence Resonance in the FitzHugh-Nagumo System	243
11.3.2	Correlation Time and Spectrum when Feedback is Applied	244
11.3.3	Control of Synchronization in Coupled FitzHugh-Nagumo Systems	245
11.3.4	What can We Control in an Excitable System?	246
11.4	Delayed Feedback Control of Noise-Induced Pulses in a Model of an Excitable Medium	247
11.4.1	Model Description	247
11.4.2	Characteristics of Noise-Induced Patterns	249
11.4.3	Control of Noise-Induced Patterns	251
11.4.4	Mechanisms of Delayed Feedback Control of the Excitable Medium	253
11.4.5	What Can Be Controlled in an Excitable Medium?	254
11.5	Delayed Feedback Control of Noise-Induced Patterns in a Globally Coupled Reaction-Diffusion Model	255
11.5.1	Spatiotemporal Dynamics in the Uncontrolled Deterministic System	256
11.5.2	Noise-Induced Patterns in the Uncontrolled System	258
11.5.3	Time-Delayed Feedback Control of Noise-Induced Patterns	260

11.5.4	Linear Modes of the Inhomogeneous Fixed Point	264
11.5.5	Delay-Induced Oscillatory Patterns	268
11.5.6	What Can Be Controlled in a Globally Coupled Reaction–Diffusion System?	269
11.6	Summary and Conclusions	270
	Acknowledgments	270
	References	270
<b>12</b>	<b>Controlling Coherence of Noisy and Chaotic Oscillators by Delayed Feedback</b>	275
	<i>Denis Goldobin, Michael Rosenblum, and Arkady Pikovsky</i>	
12.1	Control of Coherence: Numerical Results	276
12.1.1	Noisy Oscillator	276
12.1.2	Chaotic Oscillator	277
12.1.3	Enhancing Phase Synchronization	279
12.2	Theory of Coherence Control	279
12.2.1	Basic Phase Model	279
12.2.2	Noise-Free Case	280
12.2.3	Gaussian Approximation	280
12.2.4	Self-Consistent Equation for Diffusion Constant	282
12.2.5	Comparison of Theory and Numerics	283
12.3	Control of Coherence by Multiple Delayed Feedback	283
12.4	Conclusion	288
	References	289
<b>13</b>	<b>Resonances Induced by the Delay Time in Nonlinear Autonomous Oscillators with Feedback</b>	291
	<i>Cristina Masoller</i>	
	Acknowledgment	298
	References	299
<b>Part IV</b>	<b>Communicating with Chaos, Chaos Synchronization</b>	
<b>14</b>	<b>Secure Communication with Chaos Synchronization</b>	303
	<i>Wolfgang Kinzel and Ido Kanter</i>	
14.1	Introduction	303
14.2	Synchronization of Chaotic Systems	304
14.3	Coding and Decoding Secret Messages in Chaotic Signals	309
14.4	Analysis of the Exchanged Signal	311
14.5	Neural Cryptography	313
14.6	Public Key Exchange by Mutual Synchronization	315
14.7	Public Keys by Asymmetric Attractors	318
14.8	Mutual Chaos Pass Filter	319
14.9	Discussion	321
	References	323

<b>15</b>	<b>Noise Robust Chaotic Systems</b>	325
	<i>Thomas L. Carroll</i>	
15.1	Introduction	325
15.2	Chaotic Synchronization	326
15.3	2-Frequency Self-Synchronizing Chaotic Systems	326
15.3.1	Simple Maps	326
15.4	2-Frequency Synchronization in Flows	329
15.4.1	2-Frequency Additive Rössler	329
15.4.2	Parameter Variation and Periodic Orbits	332
15.4.3	Unstable Periodic Orbits	333
15.4.4	Floquet Multipliers	334
15.4.5	Linewidths	335
15.5	Circuit Experiments	336
15.5.1	Noise Effects	338
15.6	Communication Simulations	338
15.7	Multiplicative Two-Frequency Rössler Circuit	341
15.8	Conclusions	346
	<i>References</i>	346
<b>16</b>	<b>Nonlinear Communication Strategies</b>	349
	<i>Henry D. I. Abarbanel</i>	
16.1	Introduction	349
16.1.1	Secrecy, Encryption, and Security?	350
16.2	Synchronization	351
16.3	Communicating Using Chaotic Carriers	353
16.4	Two Examples from Optical Communication	355
16.4.1	Rare-Earth-Doped Fiber Amplifier Laser	355
16.4.2	Time Delay Optoelectronic Feedback Semiconductor Laser	357
16.5	Chaotic Pulse Position Communication	359
16.6	Why Use Chaotic Signals at All?	362
16.7	Undistorting the Nonlinear Effects of the Communication Channel	363
16.8	Conclusions	366
	<i>References</i>	367
<b>17</b>	<b>Synchronization and Message Transmission for Networked Chaotic Optical Communications</b>	369
	<i>K. Alan Shore, Paul S. Spencer, and Illestyn Pierce</i>	
17.1	Introduction	369
17.2	Synchronization and Message Transmission	370
17.3	Networked Chaotic Optical Communication	372
17.3.1	Chaos Multiplexing	373
17.3.2	Message Relay	373
17.3.3	Message Broadcasting	374

17.4	Summary	376
	Acknowledgments	376
	References	376
<b>18</b>	<b>Feedback Control Principles for Phase Synchronization</b>	379
	<i>Vladimir N. Belykh, Grigory V. Osipov, and Jürgen Kurths</i>	
18.1	Introduction	379
18.2	General Principles of Automatic Synchronization	381
18.3	Two Coupled Poincaré Systems	384
18.4	Coupled van der Pol and Rössler Oscillators	386
18.5	Two Coupled Rössler Oscillators	389
18.6	Coupled Rössler and Lorenz Oscillators	391
18.7	Principles of Automatic Synchronization in Networks of Coupled Oscillators	393
18.8	Synchronization of Locally Coupled Regular Oscillators	395
18.9	Synchronization of Locally Coupled Chaotic Oscillators	397
18.10	Synchronization of Globally Coupled Chaotic Oscillators	399
18.11	Conclusions	401
	References	401
<b>Part V</b>	<b>Applications to Optics</b>	
<b>19</b>	<b>Controlling Fast Chaos in Optoelectronic Delay Dynamical Systems</b>	407
	<i>Lucas Illing, Daniel J. Gauthier, and Jonathan N. Blakely</i>	
19.1	Introduction	407
19.2	Control-Loop Latency: A Simple Example	408
19.3	Controlling Fast Systems	412
19.4	A Fast Optoelectronic Chaos Generator	415
19.5	Controlling the Fast Optoelectronic Device	419
19.6	Outlook	423
	Acknowledgment	424
	References	424
<b>20</b>	<b>Control of Broad-Area Laser Dynamics with Delayed Optical Feedback</b>	427
	<i>Nicoleta Gaciu, Edeltraud Gehrig, and Ortwin Hess</i>	
20.1	Introduction: Spatiotemporally Chaotic Semiconductor Lasers	427
20.2	Theory: Two-Level Maxwell-Bloch Equations	429
20.3	Dynamics of the Solitary Laser	432
20.4	Detection of Spatiotemporal Complexity	433
20.4.1	Reduction of the Number of Modes by Coherent Injection	433
20.4.2	Pulse-Induced Mode Synchronization	435
20.5	Self-Induced Stabilization and Control with Delayed Optical Feedback	438

20.5.1	Influence of Delayed Optical Feedback	439
20.5.2	Influence of the Delay Time	440
20.5.3	Spatially Structured Delayed Optical Feedback Control	444
20.5.4	Filtered Spatially Structured Delayed Optical Feedback	449
20.6	Conclusions	451
	References	453
<b>21</b>	<b>Noninvasive Control of Semiconductor Lasers by Delayed Optical Feedback</b>	<b>455</b>
	<i>Hans-Jürgen Wünsche, Sylvia Schikora, and Fritz Henneberger</i>	
21.1	The Role of the Optical Phase	456
21.2	Generic Linear Model	459
21.3	Generalized Lang-Kobayashi Model	461
21.4	Experiment	462
21.4.1	The Integrated Tandem Laser	463
21.4.2	Design of the Control Cavity	464
21.4.3	Maintaining Resonance	465
21.4.4	Latency and Coupling Strength	465
21.4.5	Results of the Control Experiment	466
21.5	Numerical Simulation	468
21.5.1	Traveling-Wave Model	468
21.5.2	Noninvasive Control Beyond a Hopf Bifurcation	470
21.5.3	Control Dynamics	470
21.5.4	Variation of the Control Parameters	471
21.6	Conclusions	473
	Acknowledgment	473
	References	473
<b>22</b>	<b>Chaos and Control in Semiconductor Lasers</b>	<b>475</b>
	<i>Junji Ohtsubo</i>	
22.1	Introduction	475
22.2	Chaos in Semiconductor Lasers	476
22.2.1	Laser Chaos	476
22.2.2	Optical Feedback Effects in Semiconductor Lasers	478
22.2.3	Chaotic Effects in Newly Developed Semiconductor Lasers	480
22.3	Chaos Control in Semiconductor Lasers	485
22.4	Control in Newly Developed Semiconductor Lasers	494
22.5	Conclusions	497
	References	498

<b>23</b>	<b>From Pattern Control to Synchronization: Control Techniques in Nonlinear Optical Feedback Systems</b>	<b>501</b>
	<i>Björn Gütlich and Cornelia Denz</i>	
23.1	Control Methods for Spatiotemporal Systems	502
23.2	Optical Single-Feedback Systems	503
23.2.1	A Simplified Single-Feedback Model System	504
23.2.2	The Photorefractive Single-Feedback System – Coherent Nonlinearity	506
23.2.3	Theoretical Description of the Photorefractive Single-Feedback System	508
23.2.4	Linear Stability Analysis	509
23.2.5	The LCLV Single-Feedback System – Incoherent Nonlinearity	510
23.2.6	Phase-Only Mode	511
23.2.7	Polarization Mode	513
23.2.8	Dissipative Solitons in the LCLV Feedback System	513
23.3	Spatial Fourier Control	514
23.3.1	Experimental Determination of Marginal Instability	516
23.3.2	Stabilization of Unstable Pattern	517
23.3.3	Direct Fourier Filtering	518
23.3.4	Positive Fourier Control	518
23.3.5	Noninvasive Fourier Control	519
23.4	Real-Space Control	520
23.4.1	Invasive Forcing	520
23.4.2	Positioning of Localized States	522
23.4.3	System Homogenization	522
23.4.4	Static Positioning	523
23.4.5	Addressing and Dynamic Positioning	523
23.5	Spatiotemporal Synchronization	524
23.5.1	Spatial Synchronization of Periodic Pattern	524
23.5.2	Unidirectional Synchronization of Two LCLV Systems	525
23.5.3	Synchronization of Spatiotemporal Complexity	526
23.6	Conclusions and Outlook	527
	<i>References</i>	528
<b>Part VI</b>	<b>Applications to Electronic Systems</b>	
<b>24</b>	<b>Delayed-Feedback Control of Chaotic Spatiotemporal Patterns in Semiconductor Nanostructures</b>	<b>533</b>
	<i>Eckehard Schöll</i>	
24.1	Introduction	533
24.2	Control of Chaotic Domain and Front Patterns in Superlattices	536
24.3	Control of Chaotic Spatiotemporal Oscillations in Resonant Tunneling Diodes	544
24.4	Conclusions	553
	<i>Acknowledgments</i>	554
	<i>References</i>	554

<b>25</b>	<b>Observing Global Properties of Time-Delayed Feedback Control in Electronic Circuits</b>	559
	<i>Hartmut Benner, Chol-Ung Choe, Klaus Höhne, Clemens von Loewenich, Hiroyuki Shirahama, and Wolfram Just</i>	
25.1	Introduction	559
25.2	Discontinuous Transitions for Extended Time-Delayed Feedback Control	560
25.2.1	Theoretical Considerations	560
25.2.2	Experimental Setup	561
25.2.3	Observation of Bistability	562
25.2.4	Basin of Attraction	564
25.3	Controlling Torsion-Free Unstable Orbits	565
25.3.1	Applying the Concept of an Unstable Controller	567
25.3.2	Experimental Design of an Unstable van der Pol Oscillator	567
25.3.3	Control Coupling and Basin of Attraction	569
25.4	Conclusions	572
	References	573
<b>26</b>	<b>Application of a Black Box Strategy to Control Chaos</b>	575
	<i>Achim Kittel and Martin Popp</i>	
26.1	Introduction	575
26.2	The Model Systems	575
26.2.1	Shinriki Oscillator	576
26.2.2	Mackey-Glass Type Oscillator	577
26.3	The Controller	580
26.4	Results of the Application of the Controller to the Shinriki Oscillator	582
26.4.1	Spectroscopy of Unstable Periodic Orbits	584
26.5	Results of the Application of the Controller to the Mackey-Glass Oscillator	585
26.5.1	Spectroscopy of Unstable Periodic Orbits	587
26.6	Further Improvements	589
26.7	Conclusions	589
	Acknowledgment	590
	References	590
<b>Part VII</b>	<b>Applications to Chemical Reaction Systems</b>	
<b>27</b>	<b>Feedback-Mediated Control of Hypermeandering Spiral Waves</b>	593
	<i>Jan Schlesner, Vladimir Zykov, and Harald Engel</i>	
27.1	Introduction	593
27.2	The FitzHugh-Nagumo Model	594
27.3	Stabilization of Rigidly Rotating Spirals in the Hypermeandering Regime	596

27.4	Control of Spiral Wave Location in the Hypermeandering Regime	599
27.5	Discussion	605
	References	606
<b>28</b>	<b>Control of Spatiotemporal Chaos in Surface Chemical Reactions</b>	<b>609</b>
	<i>Carsten Beta and Alexander S. Mikhailov</i>	
28.1	Introduction	609
28.2	The Catalytic CO Oxidation on Pt(110)	610
28.2.1	Mechanism	610
28.2.2	Modeling	611
28.2.3	Experimental Setup	612
28.3	Spatiotemporal Chaos in Catalytic CO Oxidation on Pt(110)	613
28.4	Control of Spatiotemporal Chaos by Global Delayed Feedback	615
28.4.1	Control of Turbulence in Catalytic CO Oxidation – Experimental	616
28.4.1.1	Control of Turbulence	617
28.4.1.2	Spatiotemporal Pattern Formation	618
28.4.2	Control of Turbulence in Catalytic CO Oxidation – Numerical Simulations	619
28.4.3	Control of Turbulence in Oscillatory Media – Theory	621
28.4.4	Time-Delay Autosynchronization	625
28.5	Control of Spatiotemporal Chaos by Periodic Forcing	628
	Acknowledgment	630
	References	630
<b>29</b>	<b>Forcing and Feedback Control of Arrays of Chaotic Electrochemical Oscillators</b>	<b>633</b>
	<i>István Z. Kiss and John L. Hudson</i>	
29.1	Introduction	633
29.2	Control of Single Chaotic Oscillator	634
29.2.1	Experimental Setup	634
29.2.2	Chaotic Ni Dissolution: Low-Dimensional, Phase Coherent Attractor	635
29.2.2.1	Unforced Chaotic Oscillator	635
29.2.2.2	Phase of the Unforced System	636
29.2.3	Forcing: Phase Synchronization and Intermittency	637
29.2.3.1	Forcing with $\Omega = \omega_0$	637
29.2.3.2	Forcing with $\Omega \neq \omega_0$	638
29.2.4	Delayed Feedback: Tracking	638
29.3	Control of Small Assemblies of Chaotic Oscillators	640
29.4	Control of Oscillator Populations	642
29.4.1	Global Coupling	642
29.4.2	Periodic Forcing of Arrays of Chaotic Oscillators	643
29.4.3	Feedback on Arrays of Chaotic Oscillators	644

29.4.4	Feedback, Forcing, and Global Coupling: Order Parameter	645
29.4.5	Control of Complexity of a Collective Signal	646
29.5	Concluding Remarks	647
	Acknowledgment	648
	References	649

## Part VIII Applications to Biology

<b>30</b>	<b>Control of Synchronization in Oscillatory Neural Networks</b>	653
	<i>Peter A. Tass, Christian Hauptmann, and Oleksandr V. Popovych</i>	
30.1	Introduction	653
30.2	Multisite Coordinated Reset Stimulation	654
30.3	Linear Multisite Delayed Feedback	662
30.4	Nonlinear Delayed Feedback	666
30.5	Reshaping Neural Networks	674
30.6	Discussion	676
	References	678
<b>31</b>	<b>Control of Cardiac Electrical Nonlinear Dynamics</b>	683
	<i>Trine Krogh-Madsen, Peter N. Jordan, and David J. Christini</i>	
31.1	Introduction	683
31.2	Cardiac Electrophysiology	684
31.2.1	Restitution and Alternans	685
31.3	Cardiac Arrhythmias	686
31.3.1	Reentry	687
31.3.2	Ventricular Tachyarrhythmias	688
31.3.3	Alternans as an Arrhythmia Trigger	688
31.4	Current Treatment of Arrhythmias	689
31.4.1	Pharmacological Treatment	689
31.4.2	Implantable Cardioverter Defibrillators	689
31.4.3	Ablation Therapy	690
31.5	Alternans Control	691
31.5.1	Controlling Cellular Alternans	691
31.5.2	Control of Alternans in Tissue	692
31.5.3	Limitations of the DFC Algorithm in Alternans Control	693
31.5.4	Adaptive DI Control	694
31.6	Control of Ventricular Tachyarrhythmias	695
31.6.1	Suppression of Spiral Waves	696
31.6.2	Antitachycardia Pacing	696
31.6.3	Unpinning Spiral Waves	698
31.7	Conclusions and Prospects	699
	References	700

<b>32</b>	<b>Controlling Spatiotemporal Chaos and Spiral Turbulence in Excitable Media</b>	<b>703</b>
	<i>Sitabhra Sinha and S. Sridhar</i>	
32.1	Introduction	703
32.2	Models of Spatiotemporal Chaos in Excitable Media	706
32.3	Global Control	708
32.4	Nonglobal Spatially Extended Control	711
32.4.1	Applying Control Over a Mesh	711
32.4.2	Applying Control Over an Array of Points	713
32.5	Local Control of Spatiotemporal Chaos	714
32.6	Discussion	716
	Acknowledgments	717
	References	718
<b>Part IX</b>	<b>Applications to Engineering</b>	
<b>33</b>	<b>Nonlinear Chaos Control and Synchronization</b>	<b>721</b>
	<i>Henri J. C. Huijberts and Henk Nijmeijer</i>	
33.1	Introduction	721
33.2	Nonlinear Geometric Control	721
33.2.1	Some Differential Geometric Concepts	722
33.2.2	Nonlinear Controllability	723
33.2.3	Chaos Control Through Feedback Linearization	728
33.2.4	Chaos Control Through Input–Output Linearization	732
33.3	Lyapunov Design	737
33.3.1	Lyapunov Stability and Lyapunov’s First Method	737
33.3.2	Lyapunov’s Direct Method	739
33.3.3	LaSalle’s Invariance Principle	741
33.3.4	Examples	742
	References	749
<b>34</b>	<b>Electronic Chaos Controllers – From Theory to Applications</b>	<b>751</b>
	<i>Maciej Ogorzałek</i>	
34.1	Introduction	751
34.1.1	Chaos Control	752
34.1.2	Fundamental Properties of Chaotic Systems and Goals of the Control	753
34.2	Requirements for Electronic Implementation of Chaos Controllers	754
34.3	Short Description of the OGY Technique	755
34.4	Implementation Problems for the OGY Method	757
34.4.1	Effects of Calculation Precision	758
34.4.2	Approximate Procedures for Finding Periodic Orbits	759
34.4.3	Effects of Time Delays	759
34.5	Occasional Proportional Feedback (Hunt’s) Controller	761

34.5.1	Improved Chaos Controller for Autonomous Circuits	763
34.6	Experimental Chaos Control Systems	765
34.6.1	Control of a Magnetoelastic Ribbon	765
34.6.2	Control of a Chaotic Laser	766
34.6.3	Chaos-Based Arrhythmia Suppression and Defibrillation	767
34.7	Conclusions	768
	References	769
<b>35</b>	<b>Chaos in Pulse-Width Modulated Control Systems</b>	<b>771</b>
	<i>Zhanybai T. Zhusubaliyev and Erik Mosekilde</i>	
35.1	Introduction	771
35.2	DC/DC Converter with Pulse-Width Modulated Control	774
35.3	Bifurcation Analysis for the DC/DC Converter with One-Level Control	778
35.4	DC/DC Converter with Two-Level Control	781
35.5	Bifurcation Analysis for the DC/DC Converter with Two-Level Control	783
35.6	Conclusions	784
	Acknowledgments	788
	References	788
<b>36</b>	<b>Transient Dynamics of Duffing System Under Time-Delayed Feedback Control: Global Phase Structure and Application to Engineering</b>	<b>793</b>
	<i>Takashi Hikihara and Kohei Yamasue</i>	
36.1	Introduction	793
36.2	Transient Dynamics of Transient Behavior	794
36.2.1	Magnetoelastic Beam and Experimental Setup	794
36.2.2	Transient Behavior	795
36.3	Initial Function and Domain of Attraction	797
36.4	Persistence of Chaos	800
36.5	Application of TD <sup>F</sup> C to Nanoengineering	803
36.5.1	Dynamic Force Microscopy and its Dynamics	803
36.5.2	Application of TD <sup>F</sup> C	805
36.5.3	Extension of Operating Range	806
36.6	Conclusions	808
	References	808