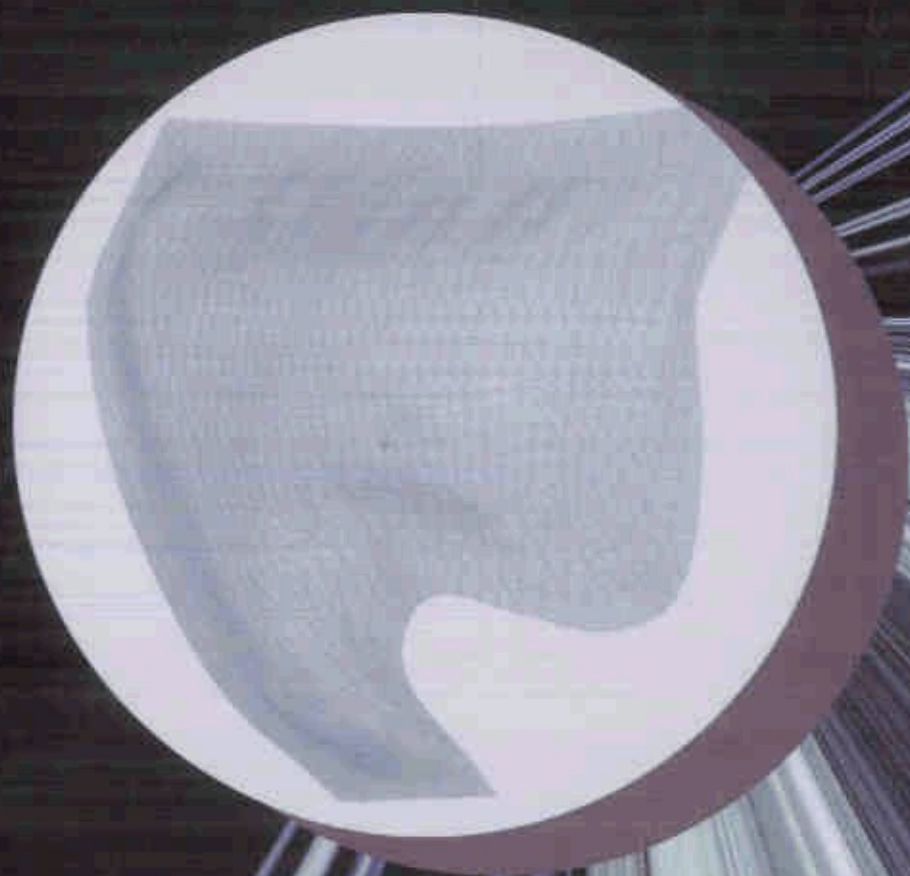


Mechanics Modeling of Sheet Metal Forming



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