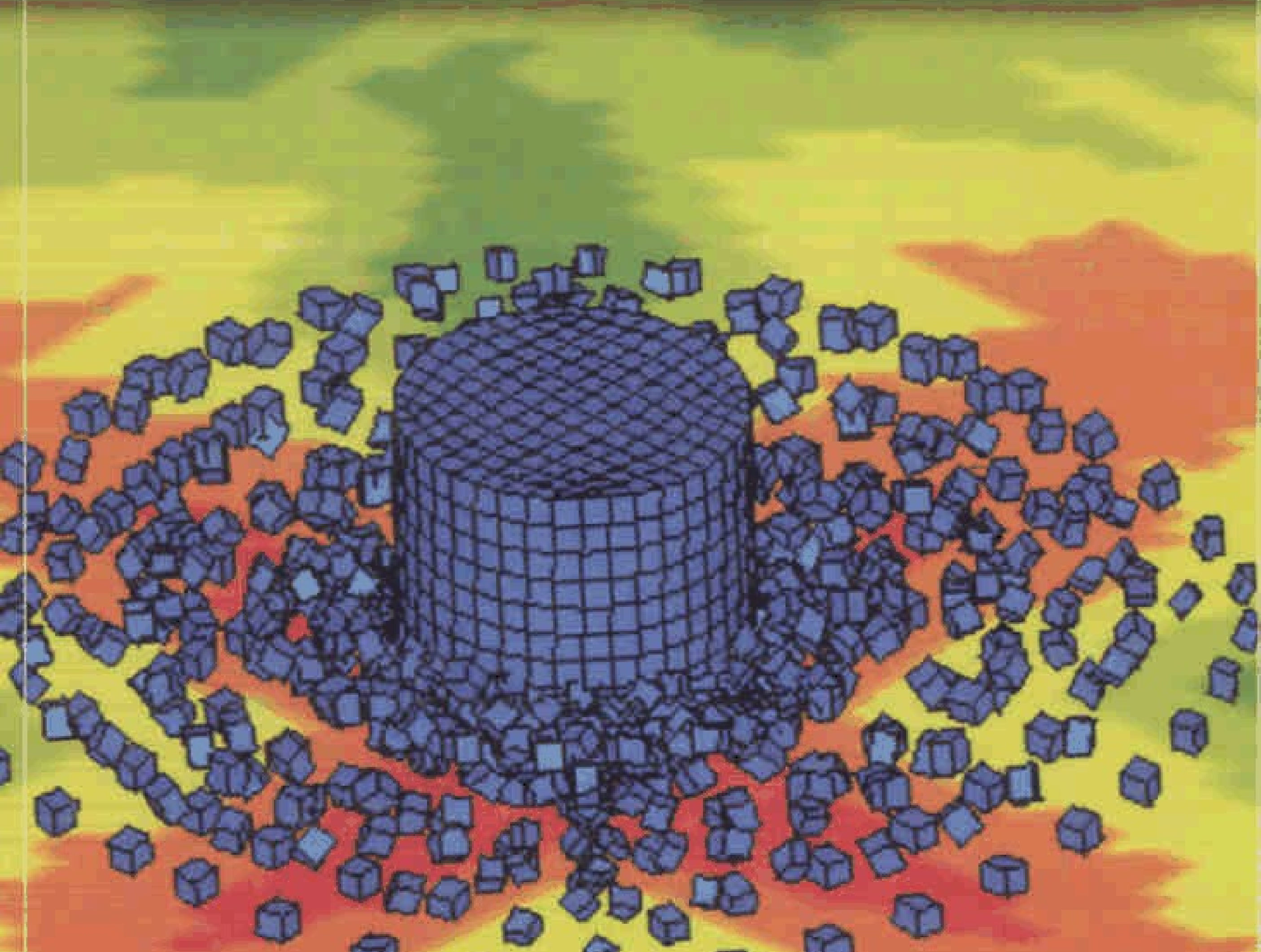
Numerical Methods for Engineers and Scientists:

An Introduction with Applications Using MATLAB



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