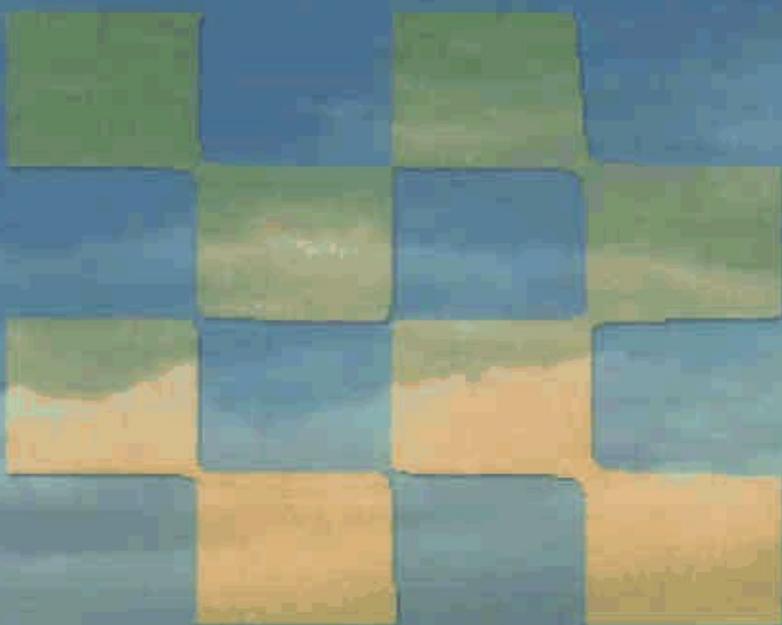


LINEAR PROGRAMMING WITH MATLAB



Michael C. Ferris
Olvi L. Mangasarian
Stephen J. Wright

MPS-SIAM Series on Optimization

Contents

Preface

xi

1	Introduction	1
1.1	An Example: The Professor's Dairy	2
1.1.1	The Setup	2
1.1.2	Formulating the Problem and a Graphical Solution	2
1.1.3	Changing the Problem	4
1.1.4	Discussion	6
1.2	Formulations	7
1.3	Applications	8
1.3.1	The Diet Problem	8
1.3.2	Linear Surface Fitting	9
1.3.3	Load Balancing Problem	10
1.3.4	Resource Allocation	10
1.3.5	Classification	11
1.3.6	Minimum-Cost Network Flow	12
1.4	Algorithms and Complexity	14
1.4.1	The Simplex Method	14
1.4.2	Interior-Point Methods	15
2	Linear Algebra: A Constructive Approach	17
2.1	Jordan Exchange	17
2.2	Linear Independence	23
2.3	Matrix Inversion	27
2.4	Exact Solution of m Equations in n Unknowns	32
2.5	Solving Linear Equations Efficiently	39
2.6	<i>LU</i> Decomposition	41
3	The Simplex Method	45
3.1	A Simple Example	46
3.2	Vertices	51
3.3	The Phase II Procedure	53
3.4	The Phase I Procedure	60
3.5	Finite Termination	65

3.5.1	The Nondegenerate Case	65
3.5.2	Cycling	66
3.5.3	The General Case	67
3.6	Linear Programs in Nonstandard Form	72
3.6.1	Transforming Constraints and Variables	72
3.6.2	Scheme I	76
3.6.3	Scheme II	80
3.6.4	Summary	86
4	Duality	89
4.1	Duality and Rank in Linear Systems	89
4.2	Duality in Linear Programming	94
4.3	Interpretation of Linear Programming Duality	96
4.4	Duality Theory	97
4.5	KKT Optimality Conditions	100
4.6	Dual Simplex Method	102
4.7	General Linear Programs	107
4.8	Big M Method	110
4.9	Applications of Duality	112
5	Solving Large Linear Programs	117
5.1	Foundations	118
5.1.1	Basic Feasible Solutions and Basis Matrices	118
5.1.2	Geometric Viewpoint	121
5.2	The Revised Simplex Method	123
5.2.1	Upper and Lower Bounds	129
5.2.2	Generating Basic Feasible Solutions	134
5.2.3	Basis Updates	139
5.2.4	Advanced Pivot Selection Mechanisms	142
5.3	Network Flow Problems	143
5.3.1	Minimum-Cost Network Flow	144
5.3.2	Shortest-Path Problem	145
5.3.3	Max-Flow Problem	146
5.3.4	Transportation Problem	147
5.3.5	Assignment Problem	149
5.3.6	Network Simplex Method	149
6	Sensitivity and Parametric Linear Programming	151
6.1	Sensitivity Analysis	151
6.2	Adding New Variables or Constraints	155
6.3	Parametric Optimization of the Objective Function	158
6.4	Parametric Optimization of the Right-Hand Side	164
7	Quadratic Programming and Complementarity Problems	169
7.1	Nonlinear Programs: Optimality Conditions	169
7.2	Quadratic Programming	172

7.2.1	Basic Existence Result	172
7.2.2	KKT Conditions	173
7.2.3	Duality	176
7.3	Linear Complementarity Problems	177
7.4	Lemke's Method	178
7.5	Bimatrix Games	185
7.5.1	Computing Nash Equilibria	186
7.5.2	Zero-Sum Games As Dual Linear Programs	192
8	Interior-Point Methods	195
8.1	Motivation and Outline	195
8.2	Newton's Method	197
8.3	Primal-Dual Methods	201
8.3.1	An Affine-Scaling Approach	202
8.3.2	Path-Following Methods	204
8.3.3	Solution of the Linear System at Each Interior-Point Iteration	208
8.3.4	Practical Primal-Dual Methods	209
8.4	Interior-Point vs. Simplex	212
8.5	Extension to Quadratic Programming	212
9	Approximation and Classification	217
9.1	Minimax Problems	217
9.2	Approximation	218
9.2.1	Chebyshev Approximation	219
9.2.2	L_1 Approximation	221
9.2.3	Approximate Solutions to Systems with Inequality Constraints	223
9.2.4	Least-Squares Problems	224
9.3	Huber Estimation	227
9.4	Classification Problems	230
A	Linear Algebra, Convexity, and Nonlinear Functions	237
A.1	Linear Algebra	237
A.2	Convex Sets	239
A.3	Smooth Functions	242
A.4	Convex Functions	242
A.5	Quadratic Functions	244
A.6	Norms and Order Notation	247
A.7	Taylor's Theorem	249
B	Summary of Available MATLAB Commands	251
B.1	Basic MATLAB Operations	251
B.2	MATLAB Functions Defined in This Book	252

Bibliography	257
Index	261