GRAVITATION AND COSMOLOGY

PRINCIPLES AND APPLICATIONS OF THE GENERAL THEORY OF RELATIVITY

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Sections marked with an asterisk are somewhat out of the book's main line of development and may be omitted in a first reading.

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