Techniques and Applications of Path Integration

L. S. SCHULMAN



Contents

PART I INTRODUCTION

1

- 1 Introducing and Defining the Path Integral, 3 Appendix: The Trotter Product Formula, 9
- 2 Probabilities and Probability Amplitudes for Paths, 13
- 3 Correspondence Limit for the Path Integral (Heuristic), 16 Appendix: Useful Integrals, 20
- 4 Vector Potentials and Another Proof of the Path Integral Formula, 22
- 5 The Ito Integral and Gauge Transformations, 27
- 6 Doing the Integral: Free Particle and Quadratic Lagrangians, 31 Appendix: Exactness of the Sum over Classical Paths, 39
- 7 Properties of Green's Functions; the Feynman-Kac Formula, 42
- 8 Functional Derivatives and Commutation Relations, 49
- 9 Brownian Motion and the Wiener Integral; Kac's Proof, 53
- 10 Perturbation Theory and Feynman Diagrams, 65

xII CONTENTS

71

PART I	I	SELECTED APPLICATIONS OF
		THE PATH INTEGRAL

- 11 Asymptotic Analysis, 73
- 12 The Calculus of Variations, 79
- 13 The WKB Approximation and its Application to the Anharmonic Oscillator, 92
- 14 Detailed Presentation of the WKB Approximation, 108
- 15 WKB Near Caustics, 118
- 16 Caustics and Uniform Asymptotic Approximations, 131
- 17 The Phase of the Semiclassical Amplitude, 143
- 18 The Semiclassical Propagator as a Function of Energy, 147
- 19 Scattering Theory, 158
- 20 Geometrical Optics, 164
- 21 The Polaron, 170
- 22 Spin and Related Matters, 182
 - 22.1 The Direct Method-Product Integrals or Time Ordered Products, 182
 - 22.2 Continuous Models for Spin, 185
- 23 Path Integrals for Multiply Connected Spaces, 190
 - 23.1 Particle Constrained to a Circle, 190
 - 23.2 Rudiments of Homotopy Theory, 197
 - 23.3 Homotopy Applied to the Path Integral, 205
 - 23.4 Extensions of Symmetric Operators, 211
- 24 Quantum Mechanics on Curved Spaces, 214

CONTENTS xili

25	Relativistic Propagators and Black Holes, 225		
26	Applications to Statistical Mechanics, 237		
27	Coherent State Representation, 242		
28	Systems with Random Impurities, 259		
29	Critical Droplets, Alias Instantons, and Metastability, 271 Appendix: Small Oscillations about the Instanton, 284		
30	Renormalization and Scaling for Critical Phenomena, 290		
31	Phase Space Path Integral, 303		
32	Omissions, Miscellany, and Prejudices, 315		
	32.1	Field Theory, 315	
	32.2	Uncompleting the Square, 325	
	32.3	Rubber: Path Integral Formulation of a Polymer as a Random Walk, 332	
	32.4	Hard Sphere Gas Second Virial Coefficient, 333	
	32.5	Adding Paths by Computer, 339	
	32.6	A Perturbation Expansion Using the Path Integral, 344	
	32.7	Solvable Path Integral with the Potential $ax^2 + b/x^2$, 344	
	32.8	Superfluidity, 346	
	32.9	Fermions, 346	
	32.10	Books and Review Papers on Path Integrals, 347	
Author Index 34			
Sul	Subject Index		
Supplements			