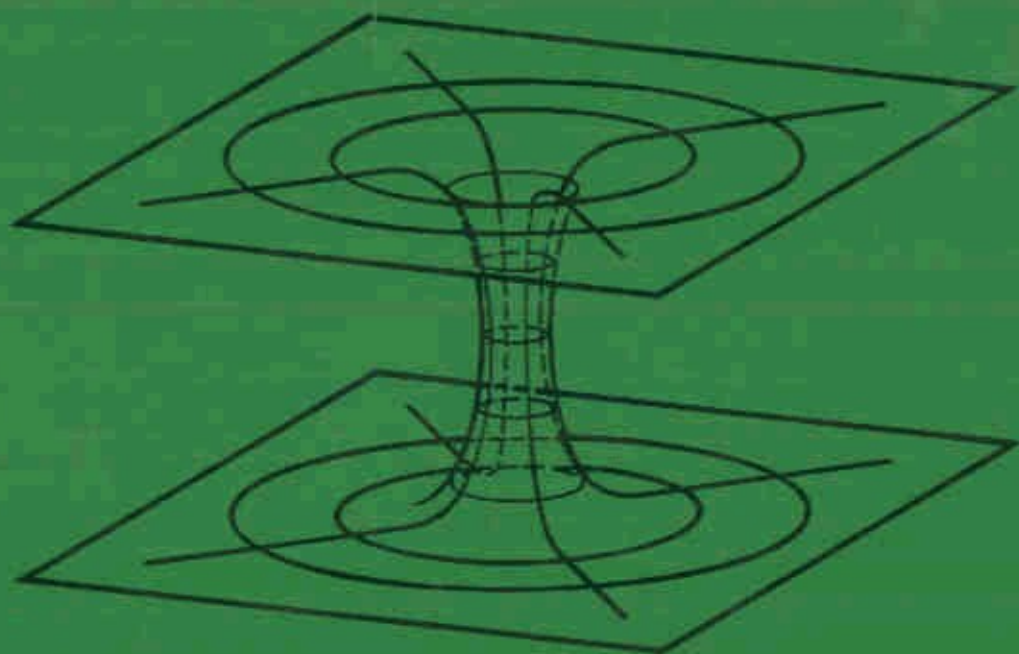

A first course in general relativity

BERNARD F. SCHUTZ



Contents



<i>Preface</i>	xi
----------------	----

1 Special relativity	1
1.1 Fundamental principles of special relativity theory (SR)	1
1.2 Definition of an inertial observer in SR	4
1.3 New units	5
1.4 Spacetime diagrams	6
1.5 Construction of the coordinates used by another observer	7
1.6 Invariance of the interval	10
1.7 Invariant hyperbolae	15
1.8 Particularly important results	18
1.9 The Lorentz transformation	24
1.10 The velocity-composition law	25
1.11 Paradoxes and physical intuition	26
1.12 Bibliography	27
1.13 Appendix	28
1.14 Exercises	30
2 Vector analysis in special relativity	36
2.1 Definition of a vector	36
2.2 Vector algebra	39
2.3 The four-velocity	44
2.4 The four-momentum	45
2.5 Scalar product	47

2.6	Applications	50
2.7	Photons	52
2.8	Bibliography	53
2.9	Exercises	54
3	Tensor analysis in special relativity	60
3.1	The metric tensor	60
3.2	Definition of tensors	61
3.3	The $\binom{0}{1}$ tensors: one-forms	62
3.4	The $\binom{0}{2}$ tensors	71
3.5	Metric as a mapping of vectors into one-forms	73
3.6	Finally: $\binom{M}{N}$ tensors	77
3.7	Index 'raising' and 'lowering'	78
3.8	Differentiation of tensors	80
3.9	Bibliography	81
3.10	Exercises	81
4	Perfect fluids in special relativity	89
4.1	Fluids	89
4.2	Dust: The number-flux vector $\tilde{N}$	90
4.3	One-forms and surfaces	94
4.4	Dust again: The stress-energy tensor	97
4.5	General fluids	99
4.6	Perfect fluids	106
4.7	Importance for general relativity	110
4.8	Gauss' law	111
4.9	Bibliography	112
4.10	Exercises	113
5	Preface to curvature	118
5.1	On the relation of gravitation to curvature	118
5.2	Tensor algebra in polar coordinates	126
5.3	Tensor calculus in polar coordinates	133
5.4	Christoffel symbols and the metric	140
5.5	The tensorial nature of $\Gamma^{\alpha}_{\beta\mu}$	143
5.6	Noncoordinate bases	144
5.7	Looking ahead	147
5.8	Bibliography	148
5.9	Exercises	148
6	Curved manifolds	151
6.1	Differentiable manifolds and tensors	151
6.2	Riemannian manifolds	154

6.3	Covariant differentiation	160
6.4	Parallel-transport, geodesics and curvature	163
6.5	The curvature tensor	167
6.6	Bianchi identities; Ricci and Einstein tensors	173
6.7	Curvature in perspective	175
6.8	Bibliography	176
6.9	Exercises	176
7	Physics in a curved spacetime	182
7.1	The transition from differential geometry to gravity	182
7.2	Physics in slightly curved spacetimes	185
7.3	Curved intuition	188
7.4	Conserved quantities	189
7.5	Bibliography	191
7.6	Exercises	191
8	The Einstein field equations	195
8.1	Purpose and justification of the field equations	195
8.2	Einstein's equations	199
8.3	Einstein's equations for weak gravitational fields	200
8.4	Newtonian gravitational fields	205
8.5	Bibliography	208
8.6	Exercises	209
9	Gravitational radiation	214
9.1	The propagation of gravitational waves	214
9.2	The detection of gravitational waves	221
9.3	The generation of gravitational waves	226
9.4	The energy carried away by gravitational waves	234
9.5	Bibliography	242
9.6	Exercises	243
10	Spherical solutions for stars	251
10.1	Coordinates for spherically symmetric spacetimes	251
10.2	Static spherically symmetric spacetimes	253
10.3	Static perfect fluid Einstein equations	255
10.4	The exterior geometry	257
10.5	The interior structure of the star	258
10.6	Exact interior solutions	261
10.7	Realistic stars and gravitational collapse	264
10.8	Bibliography	270
10.9	Exercises	271

11 Schwarzschild geometry and black holes	275
11.1 Trajectories in the Schwarzschild spacetime	275
11.2 Nature of the surface $r = 2M$	288
11.3 More-general black holes	294
11.4 Quantum mechanical emission of radiation by black holes: The Hawking process	305
11.5 Bibliography	310
11.6 Exercises	311
 12 Cosmology	 318
12.1 What is cosmology?	318
12.2 General-relativistic cosmological models	322
12.3 Cosmological observations	329
12.4 Physical cosmology	334
12.5 Bibliography	338
12.6 Exercises	338
 Appendix A:	
Summary of linear algebra	342
 Appendix B:	
Hints and solutions to selected exercises	346
 References	 359
 Index	 367