

QUANTUM FIELD THEORY

Second edition

LEWIS H. RYDER

Contents

<i>Preface to the first edition</i>	xvii
<i>Preface to the second edition</i>	xix
1 Introduction: synopsis of particle physics	1
1.1 Quantum field theory	1
1.2 Gravitation	2
1.3 Strong interactions	3
1.4 Weak interactions	4
1.5 Leptonic quantum numbers	5
1.6 Hadronic quantum numbers	7
1.7 Resonances	8
1.8 The quark model	9
1.9 $SU(2)$, $SU(3)$, $SU(4)$, ...	12
1.10 Dynamical evidence for quarks	15
1.11 Colour	18
1.12 QCD	22
1.13 Weak interactions	23
Guide to further reading	24
2 Single-particle relativistic wave equations	25
2.1 Relativistic notation	25
2.2 Klein–Gordon equation	27
2.3 Dirac equation	29
$SU(2)$ and the rotation group	30
$SL(2, C)$ and the Lorentz group	36
2.4 Prediction of antiparticles	42
2.5 Construction of Dirac spinors: algebra of γ matrices	46
2.6 Non-relativistic limit and the electron magnetic moment	52
2.7 The relevance of the Poincaré group: spin operators and the zero mass limit	55
2.8 Maxwell and Proca equations	64
2.9 Maxwell’s equations and differential geometry	69
Summary	77
Guide to further reading	77

Contents

3 Lagrangian formulation, symmetries and gauge fields	79
3.1 Lagrangian formulation of particle mechanics	80
3.2 The real scalar field: variational principle and Noether's theorem	81
3.3 Complex scalar fields and the electromagnetic field	90
3.4 Topology and the vacuum: the Bohm–Aharonov effect	98
3.5 The Yang–Mills field	105
3.6 The geometry of gauge fields	112
Summary	124
Guide to further reading	125
4 Canonical quantisation and particle interpretation	126
4.1 The real Klein–Gordon field	126
4.2 The complex Klein–Gordon field	135
4.3 The Dirac field	137
4.4 The electromagnetic field	140
Radiation gauge quantisation	142
Lorentz gauge quantisation	145
4.5 The massive vector field	150
Summary	152
Guide to further reading	153
5 Path integrals and quantum mechanics	154
5.1 Path-integral formulation of quantum mechanics	154
5.2 Perturbation theory and the S matrix	161
5.3 Coulomb scattering	170
5.4 Functional calculus: differentiation	172
5.5 Further properties of path integrals	174
Appendix: some useful integrals	179
Summary	181
Guide to further reading	181
6 Path-integral quantisation and Feynman rules: scalar and spinor fields	182
6.1 Generating functional for scalar fields	182
6.2 Functional integration	186
6.3 Free particle Green's functions	189
6.4 Generating functionals for interacting fields	196

Contents

6.5	ϕ^4 theory	200
	Generating functional	200
	2-point function	202
	4-point function	204
6.6	Generating functional for connected diagrams	207
6.7	Fermions and functional methods	210
6.8	The S matrix and reduction formula	217
6.9	Pion–nucleon scattering amplitude	224
6.10	Scattering cross section	232
	Summary	238
	Guide to further reading	239
7	Path-integral quantisation: gauge fields	240
7.1	Propagators and gauge conditions in QED	240
	Photon propagator – canonical formalism	240
	Photon propagator – path-integral method	242
	Gauge-fixing terms	242
	Propagator for transverse photons	243
7.2	Non-Abelian gauge fields and the Faddeev–Popov method	245
	Feynman rules in the Lorentz gauge	250
	Gauge-field propagator in the axial gauge	254
7.3	Self-energy operator and vertex function	255
	Geometrical interpretation of the Legendre transformation	260
	Thermodynamic analogy	262
7.4	Ward–Takahashi identities in QED	263
7.5	Becchi–Rouet–Stora transformation	270
7.6	Slavnov–Taylor identities	273
7.7	A note on ghosts and unitarity	276
	Summary	280
	Guide to further reading	281
8	Spontaneous symmetry breaking and the Weinberg–Salam model	282
8.1	What is the vacuum?	282
8.2	The Goldstone theorem	287
8.3	Spontaneous breaking of gauge symmetries	293
8.4	Superconductivity	296
8.5	The Weinberg–Salam model	298
	Summary	306
	Guide to further reading	307

Contents

9 Renormalisation	308
9.1 Divergences in ϕ^4 theory	308
Dimensional analysis	311
9.2 Dimensional regularisation of ϕ^4 theory	313
Loop expansion	317
9.3 Renormalisation of ϕ^4 theory	318
Counter-terms	321
9.4 Renormalisation group	324
9.5 Divergences and dimensional regularisation of QED	329
9.6 1-loop renormalisation of QED	337
Anomalous magnetic moment of the electron	343
Asymptotic behaviour	345
9.7 Renormalisability of QED	347
9.8 Asymptotic freedom of Yang–Mills theories	353
9.9 Renormalisation of pure Yang–Mills theories	362
9.10 Chiral anomalies	366
Cancellation of anomalies	373
9.11 Renormalisation of Yang–Mills theories with spontaneous symmetry breakdown	375
't Hooft's gauges	375
The effective potential	377
Loop expansion of the effective potential	380
Appendix A: integration in d dimensions	382
Appendix B: the gamma function	385
Summary	387
Guide to further reading	388
10 Topological objects in field theory	390
10.1 The sine–Gordon kink	391
10.2 Vortex lines	395
10.3 The Dirac monopole	402
10.4 The 't Hooft–Polyakov monopole	406
10.5 Instantons	414
Quantum tunnelling, θ -vacua and symmetry breaking	420
Summary	424
Guide to further reading	424
11 Supersymmetry	426
11.1 Introduction	426
11.2 Lorentz transformations: Dirac, Weyl and Majorana spinors	427
Some further relations	436

Contents

11.3	Simple Lagrangian model	440
	Digression: Fierz rearrangement formula	444
11.4	Simple Lagrangian model (cont.): closure of commutation relations	446
	Mass term	450
11.5	Towards a super-Poincaré algebra	452
11.6	Superspace	459
11.7	Superfields	464
	Chiral superfield	467
11.8	Recovery of the Wess–Zumino model	470
	Appendix: some 2-spinor conventions	473
	Summary	475
	Guide to further reading	475
	<i>References</i>	476
	<i>Index</i>	482