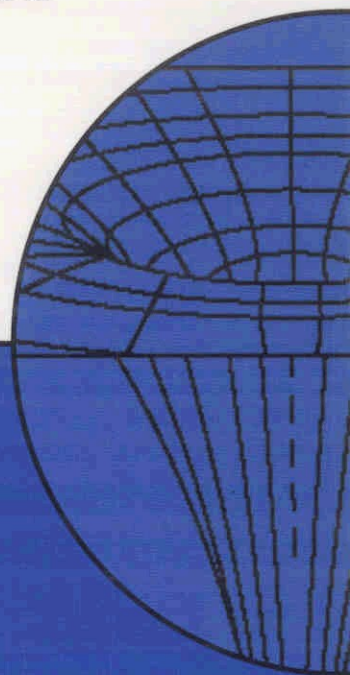


OXFORD MASTER SERIES IN PARTICLE PHYSICS,
ASTROPHYSICS, AND COSMOLOGY

Relativity, Gravitation and Cosmology

A Basic Introduction

Ta-Pei Cheng



oxford master series in condensed physics

Contents

Part I RELATIVITY Metric Description of Spacetime

1 Introduction and overview	3
1.1 Relativity as a coordinate symmetry	5
1.1.1 From Newtonian relativity to aether	5
1.1.2 Einsteinian relativity	6
1.1.3 Coordinate symmetry transformations	7
1.1.4 New kinematics and dynamics	7
1.2 GR as a gravitational field theory	8
1.2.1 Einstein's motivations for the general theory	8
1.2.2 Geometry as gravity	10
1.2.3 Mathematical language of relativity	11
1.2.4 GR is the framework for cosmology	12
Review questions	12
2 Special relativity and the flat spacetime	14
2.1 Coordinate symmetries	14
2.1.1 Rotational symmetry	14
2.1.2 Newtonian physics and Galilean symmetry	16
2.1.3 Electrodynamics and Lorentz symmetry	17
2.1.4 Velocity addition rule amended	18
2.2 The new kinematics of space and time	19
2.2.1 Relativity of spatial equilocality	20
2.2.2 Relativity of simultaneity—the new kinematics	20
2.2.3 The invariant space–time interval	22
2.3 Geometric formulation of SR	24
2.3.1 General coordinates and the metric tensor	24
2.3.2 Derivation of Lorentz transformation	28
2.3.3 The spacetime diagram	30
2.3.4 Time-dilation and length contraction	32
Review questions	35
Problems	35
3 The principle of equivalence	38
3.1 Newtonian gravitation potential—a review	38
3.2 EP introduced	39
3.2.1 Inertial mass vs. gravitational mass	40
3.2.2 EP and its significance	41

3.3	Implications of the strong EP	43
3.3.1	Gravitational redshift and time dilation	43
3.3.2	Light ray deflection calculated	48
3.3.3	Energy considerations of a gravitating light pulse	51
3.3.4	Einstein's inference of a curved spacetime	52
	Review questions	53
	Problems	53
4	Metric description of a curved space	55
4.1	Gaussian coordinates	56
4.2	Metric tensor	57
4.2.1	Geodesic as the shortest curve	59
4.2.2	Local Euclidean coordinates	61
4.3	Curvature	63
4.3.1	Gaussian curvature	63
4.3.2	Spaces with constant curvature	64
4.3.3	Curvature measures deviation from Euclidean relations	66
	Review questions	68
	Problems	69
5	GR as a geometric theory of gravity - I	71
5.1	Geometry as gravity	71
5.1.1	EP physics and a warped spacetime	73
5.1.2	Curved spacetime as gravitational field	74
5.2	Geodesic equation as GR equation of motion	75
5.2.1	The Newtonian limit	76
5.2.2	Gravitational redshift revisited	78
5.3	The curvature of spacetime	79
5.3.1	Tidal force as the curvature of spacetime	80
5.3.2	The GR field equation described	83
	Review questions	85
	Problems	85
6	Spacetime outside a spherical star	87
6.1	Description of Schwarzschild spacetime	87
6.1.1	Spherically symmetric metric tensor	88
6.1.2	Schwarzschild geometry	90
6.2	Gravitational lensing	92
6.2.1	Light ray deflection revisited	93
6.2.2	The lens equation	93
6.3	Precession of Mercury's perihelion	97
6.4	Black holes	102
6.4.1	Singularities of the Schwarzschild metric	102
6.4.2	Time measurements in the Schwarzschild spacetime	102
6.4.3	Lightcones of the Schwarzschild black hole	105

6.4.4	Orbit of an object around a black hole	108
6.4.5	Physical reality of black holes	108
	Review questions	111
	Problems	112

Part II COSMOLOGY

7	The homogeneous and isotropic universe	115
7.1	The cosmos observed	116
7.1.1	Matter distribution on the cosmic distance scale	116
7.1.2	Cosmological redshift: Hubble's law	116
7.1.3	Age of the universe	120
7.1.4	Dark matter and mass density of the universe	121
7.2	The cosmological principle	125
7.3	The Robertson–Walker metric	127
7.3.1	Proper distance in the RW geometry	129
7.3.2	Redshift and luminosity distance	130
	Review questions	133
	Problems	134
8	The expanding universe and thermal relics	136
8.1	Friedmann equations	137
8.1.1	The quasi-Newtonian interpretation	139
8.2	Time evolution of model universes	142
8.3	Big bang cosmology	145
8.3.1	Scale-dependence of radiation temperature	145
8.3.2	Different thermal equilibrium stages	147
8.4	Primordial nucleosynthesis	149
8.5	Photon decoupling and the CMB	152
8.5.1	Universe became transparent to photons	153
8.5.2	The discovery of CMB radiation	154
8.5.3	Photons, neutrinos, and the radiation–matter equality time	155
8.5.4	CMB temperature fluctuation	159
	Review questions	162
	Problems	163
9	Inflation and the accelerating universe	165
9.1	The cosmological constant	166
9.1.1	Vacuum-energy as source of gravitational repulsion	167
9.1.2	The static universe	168
9.2	The inflationary epoch	170
9.2.1	Initial conditions for the standard big bang model	171

9.2.2	The inflation scenario	173
9.2.3	Inflation and the conditions it left behind	175
9.3	CMB anisotropy and evidence for $k = 0$	178
9.3.1	Three regions of the angular power spectrum	179
9.3.2	The primary peak and spatial geometry of the universe	181
9.4	The accelerating universe in the present epoch	183
9.4.1	Distant supernovae and the 1998 discovery	184
9.4.2	Transition from deceleration to acceleration	187
9.5	The concordant picture	189
	Review questions	193
	Problems	193

Part III RELATIVITY Full Tensor Formulation

10	Tensors in special relativity	197
10.1	General coordinate systems	197
10.2	Four-vectors in Minkowski spacetime	200
10.3	Manifestly covariant formalism for E&M	205
10.3.1	The electromagnetic field tensor	205
10.3.2	Electric charge conservation	208
10.4	Energy–momentum tensors	208
	Review questions	213
	Problems	213
11	Tensors in general relativity	215
11.1	Derivatives in a curved space	215
11.1.1	General coordinate transformations	216
11.1.2	Covariant differentiation	218
11.1.3	Christoffel symbols and metric tensor	220
11.2	Parallel transport	222
11.2.1	Component changes under parallel transport	222
11.2.2	The geodesic as the straightest possible curve	224
11.3	Riemannian curvature tensor	225
11.3.1	The curvature tensor in an n -dimensional space	226
11.3.2	Symmetries and contractions of the curvature tensor	228
	Review questions	230
	Problems	231
12	GR as a geometric theory of gravity - II	233
12.1	The principle of general covariance	233
12.1.1	Geodesic equation from SR equation of motion	235

12.2	Einstein field equation	236
12.2.1	Finding the relativistic gravitational field equation	236
12.2.2	Newtonian limit of the Einstein equation	237
12.3	The Schwarzschild exterior solution	239
12.4	The Einstein equation for cosmology	244
12.4.1	Solution for a homogeneous and isotropic 3D space	244
12.4.2	Friedmann equations	246
12.4.3	Einstein equation with a cosmological constant term	247
	Review questions	248
	Problems	248
13	Linearized theory and gravitational waves	250
13.1	The linearized Einstein theory	251
13.1.1	The coordinate change called gauge transformation	252
13.1.2	The wave equation in the Lorentz gauge	253
13.2	Plane waves and the polarization tensor	254
13.3	Gravitational wave detection	255
13.3.1	Effect of gravitational waves on test particles	255
13.3.2	Gravitational wave interferometers	257
13.4	Evidence for gravitational wave	259
13.4.1	Energy flux in linearized gravitational waves	260
13.4.2	Emission of gravitational radiation	262
13.4.3	Binary pulsar PSR 1913+16	264
	Review questions	268
	Problems	269
A	Supplementary notes	271
A.1	The twin paradox (Section 2.3.4)	271
A.2	A glimpse of advanced topics in black hole physics (Section 6.4)	275
A.3	False vacuum and hidden symmetry (Section 9.2.2)	279
A.4	The problem of quantum vacuum energy as Λ (Section 9.4)	280
B	Answer keys to review questions	283
C	Solutions of selected problems	293
	References	330
	Bibliography	333
	Index	335