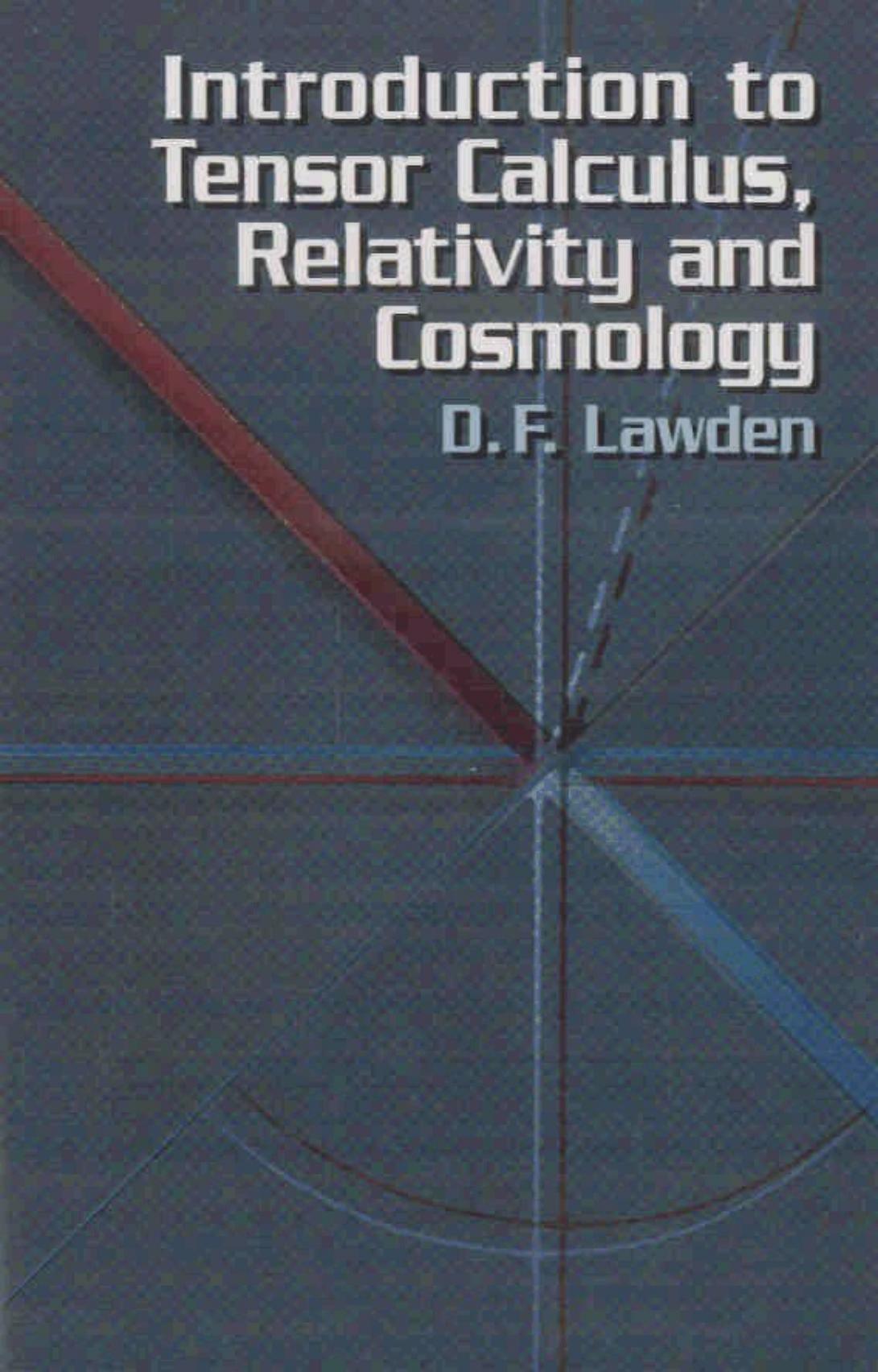


# **Introduction to Tensor Calculus, Relativity and Cosmology**

**D. F. Lawden**



# Contents

<b>Preface . . . . .</b>	<b>ix</b>
<b>List of Constants . . . . .</b>	<b>xiii</b>
<b>Chapter 1 Special Principle of Relativity. Lorentz Transformations . . . . .</b>	<b>1</b>
1. Newton's laws of motion . . . . .	1
2. Covariance of the laws of motion . . . . .	3
3. Special principle of relativity . . . . .	4
4. Lorentz transformations. Minkowski space-time . . . . .	6
5. The special Lorentz transformation . . . . .	9
6. Fitzgerald contraction. Time dilation . . . . .	12
7. Spacelike and timelike intervals. Light cone . . . . .	14
Exercises 1 . . . . .	17
<b>Chapter 2 Orthogonal Transformations. Cartesian Tensors . . . . .</b>	<b>21</b>
8. Orthogonal transformations . . . . .	21
9. Repeated-index summation convention . . . . .	23
10. Rectangular Cartesian tensors . . . . .	24
11. Invariants. Gradients. Derivatives of tensors . . . . .	27
12. Contraction. Scalar product. Divergence . . . . .	28
13. Pseudotensors . . . . .	29
14. Vector products. Curl . . . . .	30
Exercises 2 . . . . .	31
<b>Chapter 3 Special Relativity Mechanics . . . . .</b>	<b>39</b>
15. The velocity vector . . . . .	39
16. Mass and momentum . . . . .	41
17. The force vector. Energy . . . . .	44
18. Lorentz transformation equations for force . . . . .	46
19. Fundamental particles. Photon and neutrino . . . . .	47
20. Lagrange's and Hamilton's equations . . . . .	48
21. Energy-momentum tensor . . . . .	50
22. Energy-momentum tensor for a fluid . . . . .	53
23. Angular momentum . . . . .	57
Exercises 3 . . . . .	59

<b>Chapter 4 Special Relativity Electrodynamics . . . . .</b>	73
24. 4-Current density . . . . .	73
25. 4-Vector potential . . . . .	74
26. The field tensor . . . . .	75
27. Lorentz transformations of electric and magnetic vectors . . . . .	77
28. The Lorentz force . . . . .	79
29. The energy-momentum tensor for an electromagnetic field . . . . .	79
Exercises 4 . . . . .	82
<b>Chapter 5 General Tensor Calculus. Riemannian Space . . . . .</b>	86
30. Generalized $N$ -dimensional spaces . . . . .	86
31. Contravariant and covariant tensors . . . . .	89
32. The quotient theorem. Conjugate tensors . . . . .	94
33. Covariant derivatives. Parallel displacement. Affine connection . . . . .	95
34. Transformation of an affinity . . . . .	98
35. Covariant derivatives of tensors . . . . .	100
36. The Riemann-Christoffel curvature tensor . . . . .	102
37. Metrical connection. Raising and lowering indices . . . . .	105
38. Scalar products. Magnitudes of vectors . . . . .	107
39. Geodesic frame. Christoffel symbols . . . . .	108
40. Bianchi identity . . . . .	111
41. The covariant curvature tensor . . . . .	111
42. Divergence. The Laplacian. Einstein's tensor . . . . .	112
43. Geodesics . . . . .	114
Exercises 5 . . . . .	117
<b>Chapter 6 General Theory of Relativity . . . . .</b>	127
44. Principle of equivalence . . . . .	127
45. Metric in a gravitational field . . . . .	130
46. Motion of a free particle in a gravitational field . . . . .	133
47. Einstein's law of gravitation . . . . .	135
48. Acceleration of a particle in a weak gravitational field . . . . .	137
49. Newton's law of gravitation . . . . .	139
50. Freely falling dust cloud . . . . .	140
51. Metrics with spherical symmetry . . . . .	142
52. Schwarzschild's solution . . . . .	145
53. Planetary orbits . . . . .	147
54. Gravitational deflection of a light ray . . . . .	150
55. Gravitational displacement of spectral lines . . . . .	152
56. Maxwell's equations in a gravitational field . . . . .	154
57. Black holes . . . . .	155
58. Gravitational waves . . . . .	159
Exercises 6 . . . . .	163

<b>Chapter 7 Cosmology . . . . .</b>	174
59. Cosmological principle. Cosmical time . . . . .	174
60. Spaces of constant curvature . . . . .	176
61. The Robertson–Walker metric . . . . .	180
62. Hubble’s constant and the deceleration parameter. . . . .	181
63. Red shift of galaxies . . . . .	182
64. Luminosity distance . . . . .	183
65. Cosmic dynamics . . . . .	185
66. Model universes of Einstein and de Sitter . . . . .	188
67. Friedmann universes . . . . .	189
68. Radiation model . . . . .	193
69. Particle and event horizons . . . . .	195
Exercises 7 . . . . .	197
<b>References . . . . .</b>	199
<b>Bibliography . . . . .</b>	200
<b>Index . . . . .</b>	201