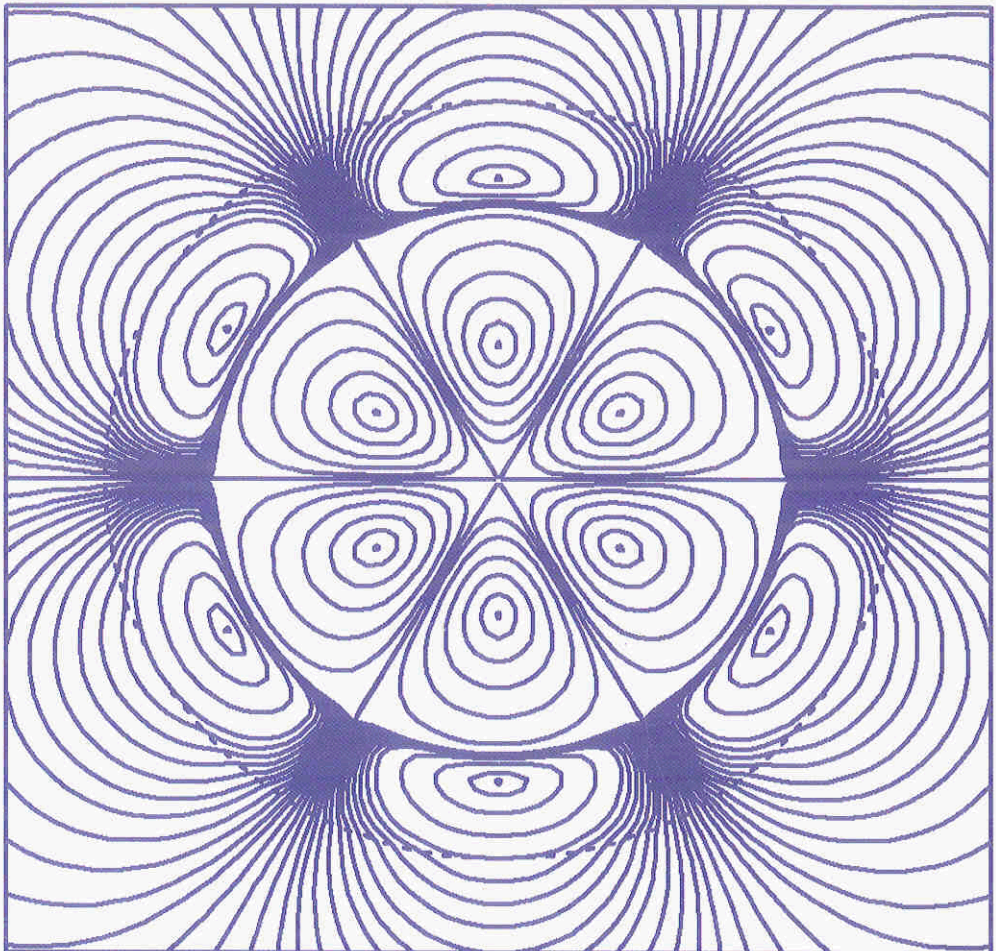


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 WILEY-VCH

# Guided Optics

Optical Fibers and All-fiber Components



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