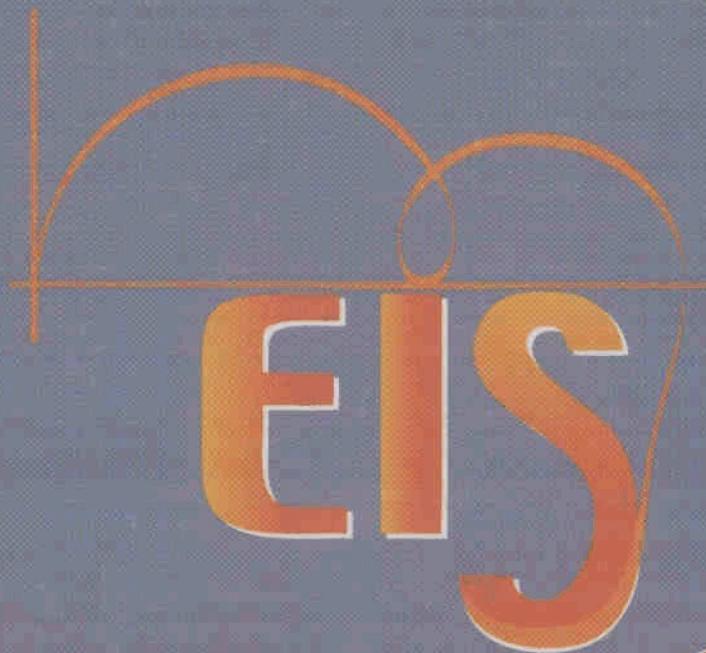




The Electrochemical Society Series

# Electrochemical Impedance Spectroscopy

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WILEY

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