



The Electrochemical Society Series

Electrochemical Impedance Spectroscopy

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A large, stylized graphic of the letters 'EIS' in a gradient of orange and red. Above the letters is a thin horizontal line with two semi-circular arcs extending upwards, resembling a Nyquist plot. The background is a dark blue oval with a fine grid pattern.

EIS

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