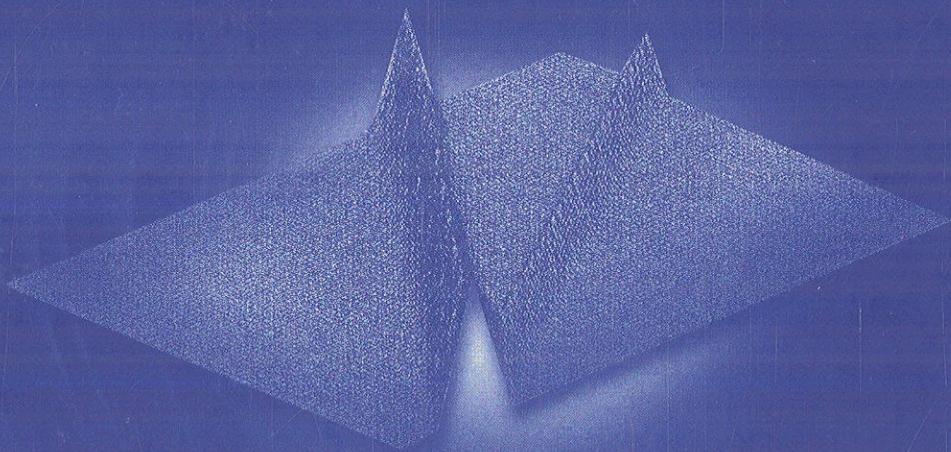


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# **WAVELET AND WAVE ANALYSIS AS APPLIED TO MATERIALS WITH MICRO OR NANOSTRUCTURE**

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**World Scientific**

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