



# Contents

<i>Preface</i>	v
1. Prologue	1
1.1 An Example . . . . .	1
1.2 Basic Definitions . . . . .	2
1.3 Notes . . . . .	9
2. Sequences	11
2.1 Lebesgue Summable Sequences . . . . .	11
2.2 Relatively Summable Sequences . . . . .	18
2.3 Uniformly Summable Sequences . . . . .	21
2.4 Properties of Univariate Sequences . . . . .	25
2.4.1 Common Sequences . . . . .	25
2.4.2 Convolution Products . . . . .	26
2.4.3 Algebraic Derivatives and Integrals . . . . .	32
2.4.4 Composition Products . . . . .	34
2.5 Properties of Bivariate Sequences . . . . .	42
2.6 Notes . . . . .	47
3. Power Series Functions	49
3.1 Univariate Power Series Functions . . . . .	49
3.2 Univariate Analytic Functions . . . . .	56
3.3 Bivariate Power Series Functions . . . . .	63
3.4 Bivariate Analytic Functions . . . . .	67
3.5 Multivariate Power Series and Analytic Functions . . . . .	68
3.6 Matrix Power Series and Analytic Functions . . . . .	71
3.7 Majorants . . . . .	72
3.8 Siegel's Lemma . . . . .	77
3.9 Notes . . . . .	82

4.	Functional Equations without Differentiation	83
4.1	Introduction . . . . .	83
4.2	Analytic Implicit Function Theorem . . . . .	86
4.3	Polynomial and Rational Functional Equations . . . . .	90
4.4	Linear Equations . . . . .	100
4.4.1	Equation I . . . . .	100
4.4.2	Equation II . . . . .	102
4.4.3	Equation III . . . . .	103
4.4.4	Equation IV . . . . .	105
4.4.5	Equation V . . . . .	107
4.4.6	Schröder and Poincaré Equations . . . . .	110
4.5	Nonlinear Equations . . . . .	114
4.6	Notes . . . . .	121
5.	Functional Equations with Differentiation	123
5.1	Introduction . . . . .	123
5.2	Linear Systems . . . . .	124
5.3	Neutral Systems . . . . .	128
5.4	Nonlinear Equations . . . . .	133
5.5	Cauchy-Kowalewski Existence Theorem . . . . .	139
5.6	Functional Equations with First Order Derivatives . . . . .	141
5.6.1	Equation I . . . . .	142
5.6.2	Equation II . . . . .	143
5.6.3	Equation III . . . . .	145
5.6.4	Equation IV . . . . .	147
5.6.5	Equation V . . . . .	148
5.6.6	Equation VI . . . . .	150
5.7	Functional Equations with Higher Order Derivatives . . . . .	152
5.7.1	Equation I . . . . .	153
5.7.2	Equation II . . . . .	154
5.7.3	Equation III . . . . .	156
5.7.4	Equation IV . . . . .	166
5.8	Notes . . . . .	170
6.	Functional Equations with Iteration	175
6.1	Equations without Derivatives . . . . .	175
6.1.1	Babbage Type Equations . . . . .	176
6.1.2	Equations Involving Several Iterates . . . . .	182
6.1.3	Equations of Invariant Curves . . . . .	190
6.2	Equations with First Order Derivatives . . . . .	197
6.2.1	Equation I . . . . .	198
6.2.2	Equation II . . . . .	202

6.2.3	Equation III . . . . .	206
6.2.4	Equation IV . . . . .	212
6.2.5	First Order Neutral Equation . . . . .	214
6.3	Equations with Second Order Derivatives . . . . .	222
6.3.1	Equation I . . . . .	223
6.3.2	Equation II . . . . .	230
6.3.3	Equation III . . . . .	235
6.3.4	Equation IV . . . . .	240
6.4	Equations with Higher Order Derivatives . . . . .	244
6.4.1	Equation I . . . . .	247
6.4.2	Equation II . . . . .	249
6.5	Notes . . . . .	257
Appendix A Univariate Sequences and Properties . . . . .		259
A.1	Common Sequences . . . . .	259
A.2	Sums and Products . . . . .	260
A.3	Quotients . . . . .	261
A.4	Algebraic Derivatives and Integrals . . . . .	261
A.5	Transformations . . . . .	262
A.6	Limiting Operations . . . . .	263
A.7	Operational Rules . . . . .	263
A.8	Knowledge Base . . . . .	266
A.9	Analytic Functions . . . . .	267
A.10	Operations for Analytic Functions . . . . .	267
<i>Bibliography</i> . . . . .		271
<i>Index</i> . . . . .		283