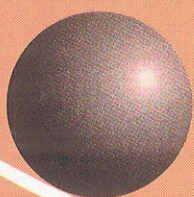


Classical Mechanics and Relativity

Harald J W Müller-Kirsten



Contents

Preface	xi
1 Introduction	1
1.1 Introduction	1
2 Recapitulation of Newtonian Mechanics	3
2.1 Introductory Remarks	3
2.2 Recapitulation of Newton's Laws	4
2.3 Further Definitions and Rotational Motion	5
2.4 Conservative Forces	7
2.5 Mechanics of a System of Particles	9
2.6 Newton's Law of Gravitation	14
2.7 Miscellaneous Examples	15
3 The Lagrange Formalism	29
3.1 Introductory Remarks	29
3.2 The Generalized Coordinates	30
3.3 The Principle of Virtual Work	32
3.4 D'Alembert's Principle, Lagrange Equations	37
3.5 Hamilton's Variational Principle, and Euler–Lagrange Equations	42
3.5.1 Hamilton's variational principle	42
3.5.2 Hamilton's principle for conservative systems	42
3.5.3 Hamilton's principle for holonomic systems	46
3.5.4 Hamilton's principle for nonholonomic systems	48
3.5.5 The general procedure	50
3.6 Symmetry Properties and Conservation Laws	51
3.7 Miscellaneous Examples	53
4 The Canonical or Hamilton Formalism	77
4.1 Introductory Remarks	77
4.2 Hamilton's Equations of Motion	78

4.3	Physical Significance of the Hamilton Function	82
4.4	<i>Variational Principle for Hamilton's Equations</i>	87
4.5	Transformation of Canonical Coordinates	88
4.6	Lagrange and Poisson Brackets	91
4.6.1	The fundamental Lagrange and Poisson brackets	91
4.6.2	Connection between Lagrange and Poisson brackets	92
4.7	The Poisson Algebra and its Significance	97
4.8	Miscellaneous Examples	100
5	Symmetries and Transformations	103
5.1	Introductory Remarks	103
5.2	Symmetries	103
5.3	The Galilei Transformation	104
5.4	Rotation and Rotation Group	109
5.4.1	Group property of coordinate transformations	109
5.4.2	The group concept	110
5.4.3	The orthogonal group $O(n)$	112
5.4.4	The groups $O(2)$ and $SO(2)$	114
5.4.5	The groups $O(3)$ and $SO(3) =: \{R\}$	116
5.4.6	The unitary groups $U(n)$ and $SU(n)$	119
5.4.7	The infinitesimal rotation of a vector	127
5.5	Rotating Reference Frames	130
5.6	Definition of Scalars, Vectors, Tensors	134
5.7	The Theorem of E. Noether	138
5.8	<i>Canonical Transformations</i>	140
5.8.1	Generators of canonical transformations	141
5.8.2	Invariance of Poisson brackets	147
5.9	Conserved Quantities	152
5.9.1	Infinitesimal canonical transformations	154
5.9.2	Infinitesimal transformations and Poisson brackets	155
5.9.3	Angular momenta and Poisson brackets	158
5.10	Miscellaneous Examples	160
6	Looking Beyond Classical Mechanics	167
6.1	Introductory Remarks	167
6.2	Aspects of Classical Statistics	168
6.2.1	Classical probabilities	168
6.2.2	The Liouville equation	175
6.2.3	Probable values of observables	179
6.3	Spacetime Formulations	183
6.3.1	Spacetime (Lorentz) transformations	184

6.3.2	The Poincaré group	186
6.3.3	Derivatives	189
6.4	From Particles to Fields	190
6.4.1	Euler–Lagrange equations	190
6.4.2	The Noether theorem	192
6.4.3	Curved spacetime	194
6.5	Miscellaneous Examples	197
7	Two-Body Central Forces	199
7.1	Introductory Remarks	199
7.2	Equations of Motion	200
7.3	Solution of the Equations	204
7.4	Differential Equation of the Orbit	208
7.5	The Kepler Problem	210
7.6	Tangential Equations of Orbits	216
7.7	Maxima and Minima of Velocities	224
7.8	Same Orbit, Different Forces	226
7.9	Period	227
7.10	Perihelion Precession of Mercury	228
7.11	Stability of Circular Orbits	233
7.12	Scattering in Central Force Fields	235
7.13	Miscellaneous Examples	241
8	Rigid Body Dynamics	257
8.1	Introductory Remarks	257
8.2	Moments of Inertia	258
8.3	Diagonalization and Principal Axes	265
8.3.1	The ellipsoid of inertia	265
8.3.2	Transformation to principal axes	268
8.4	The Equations of Motion	275
8.5	Miscellaneous Examples I	279
8.6	Force-free Motion	293
8.7	The Spinning Top in the Gravitational Field	302
8.8	Motion Relative to Rotations: Centrifugal and Coriolis Forces	311
8.9	Miscellaneous Examples II	321
9	Small Oscillations and Stability	333
9.1	Introductory Remarks	333
9.2	Resonance Frequencies and Normal Modes	333
9.3	Stability	342
9.4	Miscellaneous Examples	348

10 Motivation of the Theory of Relativity	353
10.1 Introductory Remarks	353
10.2 The Weak Equivalence Principle	354
10.3 Inertial Frames	358
10.4 The Strong Principle of Equivalence	359
10.5 The Fundamental Postulate	363
10.6 Curvature	365
10.7 Miscellaneous Examples	376
11 A Simple Look at Phenomenological Consequences	385
11.1 Introductory Remarks	385
11.2 Results of the Special Theory Summarized	386
11.3 Main Tests of General Relativity	387
11.3.1 The gravitational redshift	387
11.3.2 The gravitational deflection of light	387
11.3.3 The precession of the planet Mercury's perihelion	388
12 Aspects of Special Relativity	391
12.1 Introductory Remarks	391
12.2 Basics and Physical Motivation of the Lorentz Transformation	392
12.3 Active and Passive Transformations	395
12.4 Proper Time and Light Cones	400
12.5 Lorentz Indices and Transformations	406
12.5.1 Contravariant vectors and covariant vectors	407
12.5.2 Tensors	409
12.6 Lorentz Boosts in Electrodynamics	410
12.7 Curvature due to Lorentz Contraction	411
12.8 Covariantization of Newton's Equation of a Charged Particle	412
12.9 The Tangent Vector	417
12.10 Miscellaneous Examples	419
13 Equation of Motion of a Particle in a Gravitational Field	433
13.1 Introductory Remarks	433
13.2 Equation of Motion	433
13.3 Reduction to Newton's Equation	437
13.4 Rotation Observed from an Inertial Frame	440
13.5 The Redshift	443
14 Tensor Calculus for Riemann Spaces	445
14.1 Introductory Remarks	445
14.2 Tensors	446

14.3	Symmetric and Antisymmetric Tensors	449
14.4	Definition of Other Important Quantities	450
14.4.1	Transformation of the metric tensor	450
14.4.2	Pseudo-tensors and duals	450
14.4.3	Volume forms	452
14.5	Covariant Derivatives by the Method of Parallel Transport of a Vector	455
14.6	Metric Affinity and Christoffel Symbols	462
14.7	Raising and Lowering of Indices	464
14.8	Rewriting Co- and Contravariant Derivatives	465
14.9	Covariant Divergence, Rotation etc.	466
15	Einstein's Equation of the Gravitational Field	471
15.1	Introductory Remarks	471
15.2	The Riemann Curvature Tensor	471
15.3	Bianchi Identities and Ricci–Einstein Tensor	475
15.4	The Energy–Momentum Tensor	481
15.4.1	The energy–momentum tensor in electrodynamics	481
15.4.2	The general case	485
15.5	<i>Einstein's Equation of the Gravitational Field</i>	487
15.6	Newton's Potential from Einstein's Equation	491
16	The Schwarzschild Solution	497
16.1	Introductory Remarks	497
16.2	The Spherical Solution Outside the Source	499
16.3	The Schwarzschild Solution for $\Lambda = 0$	501
16.4	The Schwarzschild Solution for $\Lambda \neq 0$	507
16.5	The Relativistic Kepler Problem	511
16.6	The Light Ray in the Schwarzschild Field	516
Appendix A:	Schwarzschild Orbit Solution	525
A.1	Introductory remarks	525
A.2	The elliptic integral	526
A.3	Evaluating the elliptic integral	528
Appendix B:	Reissner–Nordstrom Metric	531
B.1	Introductory remarks	531
B.2	The metric	531
B.3	The energy-momentum tensor	535
B.4	The energy-momentum tensor for an electrostatic field	537
B.5	Christoffel symbols and Riemann tensor	539

B.6	The Einstein equation	542
B.7	Evaluating the electrostatic and gravitational fields . .	544
Bibliography		549
Index		553