

# Discrete Mathematical Structures

for Computer Scientists and Engineers

M.K. Das



Alpha  
Science



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*Acknowledgements*

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