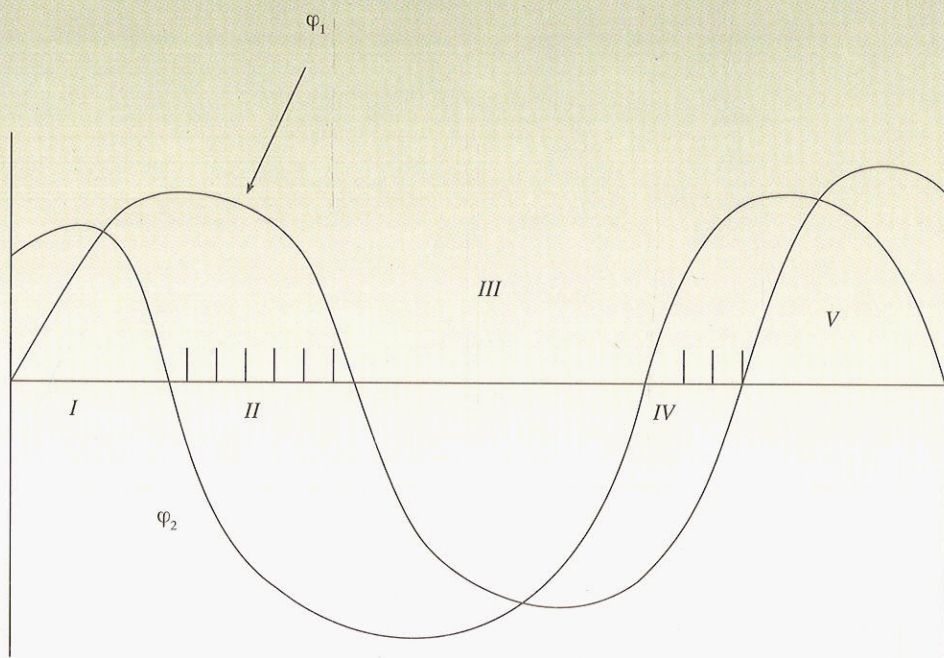


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Recent Progress in Conformal Geometry



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Contents

Preface

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