



Noise and Signal Interference in Optical Fiber Transmission Systems

An Optimum
Design Approach

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 WILEY

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