

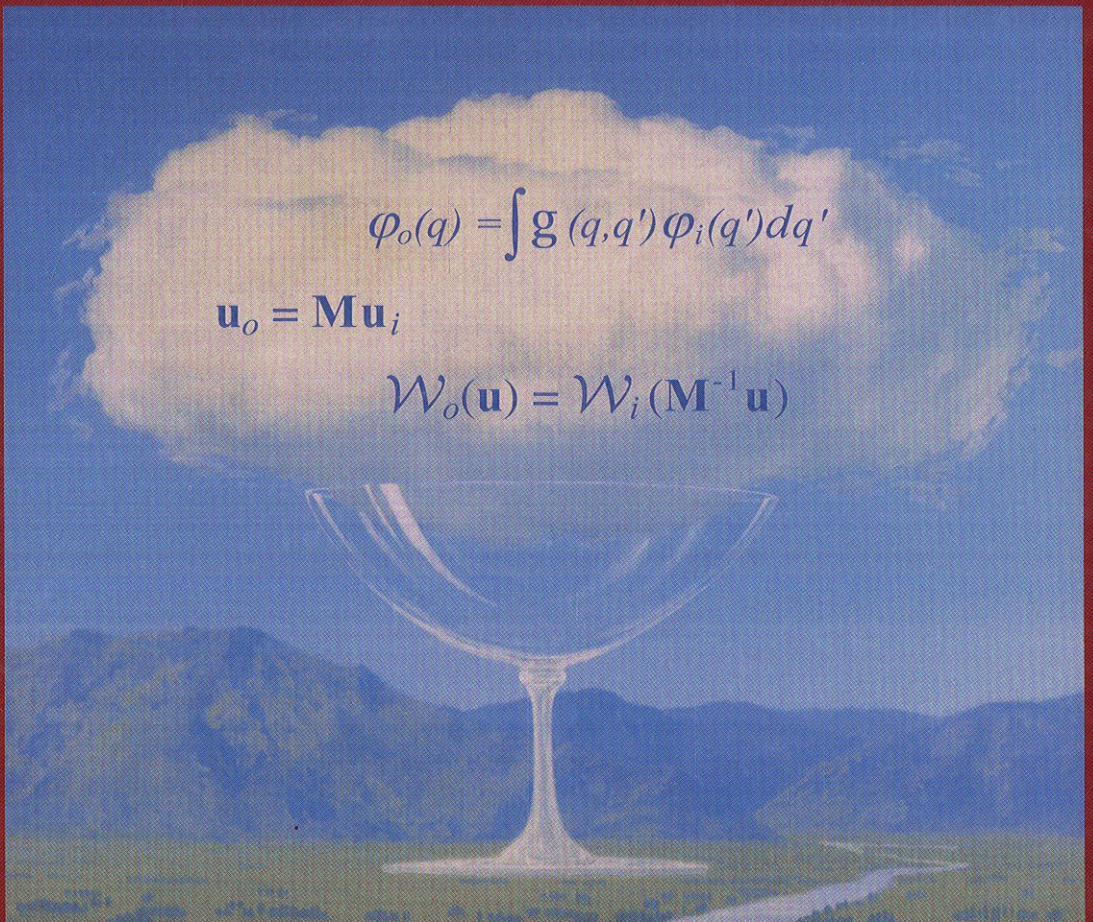


# LINEAR RAY AND WAVE OPTICS IN PHASE SPACE

$$\varphi_o(q) = \int g(q, q') \varphi_i(q') dq'$$

$$\mathbf{u}_o = \mathbf{M} \mathbf{u}_i$$

$$\mathcal{W}_o(\mathbf{u}) = \mathcal{W}_i(\mathbf{M}^{-1} \mathbf{u})$$



A. TORRE

# Contents

1. Hamiltonian Picture of Light Optics. First-Order Ray Optics	
1.1 Introduction	1
1.2 Hamiltonian picture of light-ray propagation	3
1.3 Hamiltonian picture of light-ray propagation: formal settings	9
1.4 Hamilton's equations for the light-ray	19
1.5 Lie transformations in the optical phase space	24
1.6 Linear ray optics and quadratic Hamiltonian functions	30
1.7 Planar model of first-order optical systems	36
1.8 ABCD matrix and focal, principal and nodal planes	44
1.9 Summary	53
Problems	53
References	55
2. First-Order Optical Systems: The Ray-Transfer Matrix	
2.1 Introduction	59
2.2 Ray-ensemble description of light propagation	62
2.3 Quadratic monomials and symplectic matrices	88
2.4 Quadratic monomials and first-order optical systems	93
2.5 Quadratic monomials in phase space	99
2.6 Summary	105
Problems	106
References	107
3. The Group of 1D First-Order Optical Systems	
3.1 Introduction	111
3.2 Ray matrix of composite optical systems	113
3.3 The subgroup of free propagation and thin lens matrices	115
3.4 Optical matrices factorized in terms of free-medium sections and thin lenses	120
3.5 Wei-Norman representation of optical elements: LST synthesis	131
3.6 Rotations and squeezes in the phase plane	134
3.7 Iwasawa representation of optical elements: LSF $^{\alpha}$ synthesis	151
3.8 Canonical and noncanonical representations of symplectic matrices	153
3.9 Integrating the equation for the ray transfer matrix	156
3.10 Summary	162
Problems	162
References	164
4. Wave-Optical Picture of First-Order Optical Systems	
4.1 Introduction	167
4.2 Essentials of the scalar wave model of light. The paraxial wave equation in a quadratic medium	169

4.3 Ray and wave optics	174
4.4 From the ray-optical matrix to the wave-optical operator	186
4.5 Eigenfunctions of $\hat{q}$ and $\hat{p}$ : point-like and spatial harmonic waveforms	194
4.6 Spatial Fourier representation of optical wave fields	198
4.7 Summary	214
Problems	215
References	216
<b>5. 1D First-Order Optical Systems: The Huygens-Fresnel Integral</b>	
5.1 Introduction	221
5.2 Quadratic Hamiltonians and metaplectic Lie algebra	224
5.3 Wave-optical transfer relations for an $ABCD$ system	234
5.4 The optical Fourier transform	242
5.5 Recovering the ray-optical description	257
5.6 Wave-optical propagators as unitary representations of linear canonical transformations	261
5.7 Summary	266
Problems	267
References	268
<b>6. The Wigner Distribution Function: Analytical Evaluation</b>	
6.1 Introduction	271
6.2 The optical Wigner distribution function: basic concepts	277
6.3 The Wigner distribution function: basic properties	282
6.4 The Wigner distribution function of light signals: further examples	303
6.5 Summary	333
Problems	333
References	335
<b>7. The Wigner Distribution Function: Optical Production</b>	
7.1 Introduction	341
7.2 The sliding-window Fourier transform	343
7.3 The Wigner distribution function and the general class of space-frequency signal representations	354
7.4 The ambiguity function	358
7.5 Understanding the Wigner and ambiguity functions from the viewpoint of the mutual intensity function	369
7.6 Optical production of the Wigner distribution function: general considerations	379
7.7 Wigner processor for 1D real signals: basic configurations	384
7.8 Wigner processor for 1D complex signals: basic configurations	394
7.9 The smoothed Wigner distribution function and the cross-ambiguity function: optical production	398
7.10 Summary	400
Problems	400
References	403

8. 1D First-Order Optical Systems: Transfer Laws for the Wigner Distribution Function	
8.1 Introduction	409
8.2 From the wave function to the phase-space representation	411
8.3 First-order optical systems: propagation law for the Wigner distribution function	424
8.4 The Wigner distribution function and the optical Fourier transform: linking Fourier optics to Wigner optics	438
8.5 Transport equation for the Wigner distribution function	451
8.6 Summary	456
Problems	457
References	458
9. 1D First-Order Optical Systems: Moments of the Wigner Distribution Function	
9.1 Introduction	463
9.2 Basic notions on moments	466
9.3 Preliminaries to the calculation of the moments of the Wigner distribution function	472
9.4 Wigner distribution function: local and global moments	477
9.5 Gaussian Wigner distribution functions: the variance matrix and its evolution	492
9.6 Propagation laws for the moments of the Wigner distribution function in first-order optical systems	499
9.7 Higher-order moments of the Wigner distribution function	512
9.8 Summary	514
Problems	515
References	516
A. Lie algebras and Lie groups: basic notions	519
Index	523