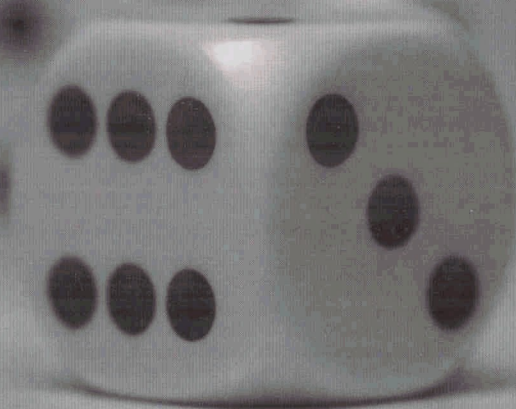


**THE FRANK J. FABOZZI SERIES**



# **BAYESIAN METHODS IN FINANCE**

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